Quantum Eather Model

Volume III: Spectral Reconstruction of Classical, Quantum, and Relativistic Physics.

Recovering canonical laws and phenomena within the Quantum Aether framework

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Contents

I Spectral Quantum Foundations **1**

1 Foundations of Spectral Quantum Mechanics: State, Measurement, and Entanglement **2**

1.1 Introduction . . . 2
1.2 Ontological Postulates and Operational Scope . . . 3
1.2.1 The spectral medium and its fields . . . 3
1.2.2 Events, instruments, and coarse description . . . 4
1.2.3 Foundational gates for the quantum sector . . . 5
1.3 From the Quadratic Spectral Action to a Hilbert Space . . . 6
1.3.1 Quadratic action and mode decomposition . . . 6
1.3.2 Symplectic form, inner product, and canonical variables . . . 7
1.3.3 Quantization and equal–time algebra . . . 8
1.3.4 Uncertainty as positivity of the quadratic form . . . 9
1.4 Events as Couplings and the Spectral Lindblad Generator . . . 9
1.4.1 Microscopic interaction Lagrangians . . . 10
1.4.2 Markovian coarse–graining and locality . . . 10
1.4.3 Causal, CP Lindblad form from spectral data . . . 11
1.4.4 Entropy production and arrow of time . . . 11
1.5 Measurement as Spectral Flooding and the Born Limit . . . 12
1.5.1 Pointer sectors and phase locking . . . 12
1.5.2 Stationary energy–share derivation of Born weights . . . 13
1.5.3 Deviations and bounds . . . 14
1.5.4 The Zeno and anti–Zeno regimes . . . 15
1.5.5 The Damascus Interpretation: Collapse as Emergent Spectral Flooding 15
1.5.6 Damascus vs. Copenhagen: a focused comparison . . . 16
1.6 Entanglement as Structural Nonseparability . . . 17
1.6.1 χ^{μ}-threads and bipartitions . . . 17
1.6.2 Bell/CHSH and no–signalling . . . 18
1.6.3 Cutting threads: degradations without signalling . . . 19
1.7 Case Studies: Double Slit and Colored Zeno . . . 20
1.7.1 Double slit with phase-only tagging . . . 20
1.7.2 Colored Zeno in waveguides / cold atoms . . . 21
1.8 Consistency Gates and Restoration . . . 22
1.8.1 Mapping to S/G/TQ/D gates . . . 23
1.8.2 Restoration lemma across scales . . . 24
1.9 Interfaces and Outlook . . . 25
1.9.1 Bridge to emergent gauge symmetries . . . 25
1.9.2 Numerical protocols and datasets . . . 25

1.9.3 Falsifiable signatures . . . 26
1.10 Notation and Parameter Map (for quick reference) . . . 27
Notation and Parameter Map . . . 27

II Classical Recovery: Mass, Gravity, and Motion 29

2 Mass, Gravity, and Worldline Dynamics 30
2.1 What is Mass? A Spectral Definition . . . 30
2.2 Emergent Gravity: Field Equations and Weak-Field Limit . . . 31
2.3 Worldline Principle and Geodesic Motion . . . 32
2.4 Finite-Size Structure: Spin and Multipoles (MPD System) . . . 33
2.5 Post-Newtonian (PN) Recovery and PPN Parameters . . . 33
2.6 Dissipative Corrections and Radiation Reaction . . . 34
2.7 Synthesis: Equivalence Principle and Classical Closure . . . 34
2.8 PPN–Spectral Parameter Map and Observational Bounds . . . 35
2.8.1 Light bending, Shapiro delay, and perihelion: closed forms . . . 35
2.9 Worked Example: Lense–Thirring Precession (Earth–LAGEOS) . . . 36
2.10 Emergent Electrodynamics from Spectral Phase . . . 36
2.10.1 Spectral U(1) and field strength . . . 36
2.10.2 Effective Maxwell equations and constitutive relations . . . 37
2.10.3 Charges and currents from solitons . . . 37
2.10.4 Lorentz force and energy–momentum exchange . . . 37
2.10.5 Wave propagation, optics, and Poynting theorem . . . 37
2.10.6 Static limits and recovery of Coulomb/Biot–Savart . . . 37
2.10.7 Radiation and reaction (GKSL-consistent) . . . 38
2.11 Conservation Laws and Continuity Equations . . . 38
2.11.1 Noether data and covariant balances . . . 38
2.11.2 3+1 split: energy and momentum balances . . . 39
2.11.3 Angular momentum and Belinfante symmetrization . . . 39
2.11.4 Charge, mass/number, and phase continuity . . . 39
2.11.5 Entropy current and the arrow of time . . . 40
2.11.6 Kubo formulas and transport from spectral correlators . . . 40
2.11.7 Compact continuity sheet (aether frame) . . . 40
2.11.8 Consistency Gates Crosswalk . . . 40
2.12 Hydrodynamics and Elasticity of the Spectral Medium . . . 41
2.12.1 From the structural Lagrangian to balance laws . . . 41
2.12.2 Anisotropic transport and positivity . . . 41
2.12.3 Linear modes: sound and shear with anisotropy . . . 42
2.12.4 Elastic limit and spectral moduli . . . 42
2.12.5 Shock discontinuities and Rankine–Hugoniot conditions . . . 43
2.12.6 Conductivity and Onsager matrix . . . 43
2.12.7 Gate summary and spectral closure . . . 43
2.13 Geometric and Wave Optics on the Emergent Metric . . . 43
2.13.1 Eikonal limit and Fermat's principle on g_{eff} . . . 43
2.13.2 Effective refractive index and birefringence . . . 44
2.13.3 Transport of amplitude and polarization . . . 44
2.13.4 Lensing and ray congruences . . . 44

2.13.5 Interfaces: anisotropic Snell–Descartes laws . . . 45
2.13.6 Caustics, envelopes, and validity . . . 45
2.13.7 Gate summary for optics . . . 45
2.14 Rigid-Body Rotation and Classical Spin . . . 45
2.14.1 Extended bodies on g_{eff} and MPD-type dynamics . . . 45
2.14.2 Rigid-body kinematics and Euler equations on g_{eff} . . . 46
2.14.3 Gyroscope precession and spin–curvature/director couplings . . . 47
2.14.4 Classical spin from internal circulation . . . 47
2.14.5 Stability, bounds, and gates . . . 47
2.15 Radiation Reaction and Consistent Damping (GKSL) . . . 48
2.15.1 Radiative channels of the spectral medium . . . 48
2.15.2 Causal memory kernel and local reduction . . . 48
2.15.3 Power balance and Larmor-type formulae . . . 49
2.15.4 GKSL limit and fluctuation–dissipation . . . 49
2.15.5 Runaway avoidance and well-posedness . . . 49
2.15.6 Illustrative closure: Landau–Lifshitz form on g_{eff} . . . 50
2.16 Hamilton–Jacobi Limit with Validity Conditions . . . 50
2.17 Targeted Numerical Matching: Three Benchmarks . . . 51
2.17.1 Free fall / perihelion (timelike rays). . . . 51
2.17.2 Light bending / Shapiro (null rays). . . . 52
2.17.3 Shear waves & anti/normal birefringence. . . . 52
2.18 Parameter Map: Spectral → Classical . . . 52
2.19 Strong Anisotropy: Birefringence Gates . . . 52
2.20 Unified Wave Energy–Momentum with Causality/Positivity . . . 53
2.21 Guide to Classical Tests (Comparison Sheet) . . . 53

3 Mass–Energy Equivalence as a Spectral Theorem 57
3.1 Definitions: Energy, Momentum, and the Emergent Light Cone . . . 57
3.2 Dispersion and the Mass–Energy Theorem . . . 59
3.3 Rest Mass of Solitons and Binding Energies . . . 61
3.4 Universality of c: RG Flow and Constraints . . . 64
3.5 Energy Balance, Radiation, and GKSL Consistency . . . 66
3.6 Tests, Calibrations, and Numerical Showcases . . . 68
3.6.1 Compton calibration and annihilation calorimetry . . . 68
3.6.2 Nuclear-scale binding as a mass defect . . . 69
3.6.3 Numerical showcase A: boosted wavepacket . . . 69
3.6.4 Numerical showcase B: head-on soliton collision . . . 70
3.6.5 Acceptance metrics and reporting standard . . . 70
3.7 Gates and Scope of Validity . . . 71

III Spectral Recovery of Classical and Quantum Physics 73

4 Precision Relativity and Classical Tests 74
4.1 Setup: Emergent Geodesics, Local Frames, and Effective Lorentz Kinematics 74
4.1.1 Equivalence principle in the emergent medium . . . 74
4.2 Post-Newtonian Expansion and Mapping to PPN . . . 75
4.3 Gravitational-Wave Speed and Ringdown . . . 75
4.4 Consistency Gates and Mapping to Spectral Parameters . . . 76

5 Emergent Electromagnetism and Energy–Momentum Forms 77
5.1 U(1) from Fiber Isometries and the Spectral Connection 77
5.2 Noether Charge and Effective Maxwell on g_{eff} 78
5.3 Constitutive Tensor, Fermat Principle, and Birefringence 78
5.4 Poynting Theorem and Stress–Energy in the Spectral Medium 79
5.5 Consistency Gates and Observational Touchstones 79

6 Scattering Theory and the S–Matrix on a Spectral Background 81
6.1 Asymptotic Regions, Wave Operators, and the S–Map 81
6.2 LSZ Reduction with Anisotropic Dispersion 81
6.3 Källén–Lehmann Representation and Microcausality 82
6.4 Optical Theorem: Unitarity and its CP Extension 82
6.5 Forward Dispersion Relations and Positivity Bounds 83
6.6 From Spectral Couplings to Cross Sections 83
6.7 Gates and Scope 83

7 Quantum Structure: Path Integrals, Coherent States, and Born as an Effective Limit 85
7.1 Hilbert Space Completion from the Quadratic Spectral Action 85
7.2 Spectral Path Integral on g_{eff} 86
7.3 Coherent States, Classicality, and Wigner Transport 86
7.4 Born Weights as a Stabilization Limit (Damascus Interpretation) 87
7.5 Spectral Path Integral for Open Dynamics: Tests 87
7.6 Gates, Restoration, and Scope 88

8 Precision Calibration: Corrected QED and Atomic Spectra 89
8.1 Effective Electrodynamics on g_{eff} and Spectral Counterterms 89
8.2 Dirac–Coulomb Bound States on g_{eff}: Fine Structure 90
8.3 Lamb Shift and Vacuum Polarization with Spectral Corrections 90
8.4 Anomalous Magnetic Moments a_e and a_μ 91
8.5 Hyperfine Structure and Sidereal Signatures 91
8.6 Rydberg Constant, α, and Consistency 91
8.7 Parameter Map and Conservative Constraints 92

9 Emergent Gauge Symmetry and BRST/FP Structure 93
9.1 From Fiber Isometries to Gauge Orbits 93
9.2 Gauge Fixing on g_{eff} and the FP Determinant 94
9.3 BRST Symmetry and Nilpotency 94
9.4 Ward/Slavnov–Taylor and Gauge–Parameter Independence 94
9.5 Background Field Method and Running Couplings 95
9.6 GKSL–Compatible Dissipation and Modified Ward Identities 95
9.7 Gates and Unitarity in the Physical Subspace 95

10 Topological Anomalies and Inflow: ABJ, 't Hooft, and WZW/CS 96
10.1 Anomaly Polynomial and Descent on g_{eff} 96
10.2 ABJ Anomaly on g_{eff} (Fujikawa) 97
10.3 Perturbative Gauge/Mixed/Gravitational Anomalies and Cancellation . . 97
10.4 't Hooft Anomaly Matching for Global Symmetries 97
10.5 WZW and CS Terms from Anomaly Inflow 98

10.6 Gates and GKSL Compatibility . . . 98
10.7 Diagnostics and Phenomenology . . . 99

11 Strong Sector: Confinement and Chiral Symmetry Breaking 100
11.1 Emergent Color and Confinement Diagnostics . . . 100
11.2 Chiral Symmetry and the Low–Energy Effective Action . . . 101
11.3 Baryons as Skyrmions and Faddeev–Skyrme Knots . . . 101
11.4 Trace Anomaly, Dilaton, and Scalar Sector . . . 102
11.5 GOR Relations from the Spectral Medium . . . 102
11.6 Hadron Spectrum: Scaling and Benchmarks . . . 102
11.7 Confinement at Finite T and the Center Symmetry . . . 103
11.8 Gates, Positivity, and Open–System Effects . . . 103
11.9 Phenomenological Program . . . 103

12 Emergent Electroweak Sector: Structural Symmetry Breaking 105
12.1 Gauge Structure from Fiber Isometries . . . 105
12.2 Structural Higgs: Order Parameter and Mass Generation . . . 105
12.3 Custodial Symmetry and the ρ Parameter . . . 106
12.4 Fermion Masses, Mixings, and Alignment . . . 106
12.5 Unitarity in VV Scattering and Higgs–like Resonance . . . 107
12.6 Precision Constraints and Spectral Mapping . . . 107
12.7 Gates: S/G/TQ/D for the Electroweak Sector . . . 107
12.8 Phenomenology Program and LEP/LHC Interfaces . . . 108

13 Flavor and Neutrinos: Spectral Origin of Generations 109
13.1 Spectral Origin of Generations . . . 109
13.1.1 Internal fiber and the spectral Dirac operator . . . 109
13.1.2 Bound-state counting: why three? . . . 110
13.2 Hierarchies and Mixing from Overlaps . . . 110
13.2.1 Yukawa matrices as overlap integrals . . . 110
13.2.2 Wolfenstein-like textures and CKM . . . 110
13.2.3 Leptons and PMNS structure . . . 111
13.3 CP Structure from Geometric Phases and WZW/CS Terms . . . 111
13.3.1 Berry phases on the fiber . . . 111
13.3.2 Jarlskog invariant and emergent CP . . . 111
13.4 Spectral See–Saw for Neutrino Masses . . . 111
13.4.1 Construction . . . 111
13.4.2 Neutrinoless double beta and phases . . . 112
13.5 Predictions and Constraints . . . 112
13.6 Consistency Gates and Anomaly Matching . . . 112
13.7 Numerical Protocol and Parameter Map . . . 112

14 Thermodynamics and Hydrodynamics of the Spectral Medium 114
14.1 Local Equilibrium, State Variables, and Balances . . . 114
14.2 Constitutive Relations with Uniaxial Anisotropy . . . 115
14.3 Entropy Current and the Dissipation Gate . . . 115
14.4 Linear Modes: Sound, Shear, and Anisotropy . . . 116
14.5 Linear Response, Kubo Formulae, and FDT . . . 116
14.6 Numerical Protocols and Validation . . . 117

15 Quantum Field Theory on an Effective Curved Spacetime 118
15.1 Free Fields on $(\mathcal{M}, g_{\text{eff}})$: Equations and Inner Product . . . 118
15.2 Vacuum Choices and the Hadamard Condition . . . 118
15.3 Particle Creation in Expanding Backgrounds . . . 119
15.4 Detector Thermality: Unruh Effect on g_{eff} . . . 119
15.5 Horizons and Hawking Radiation of the Effective Geometry . . . 119
15.6 Renormalized $\langle T_{\mu\nu} \rangle$ and Semiclassical Backreaction . . . 120
15.7 Open-System Extensions: Covariant GKSL on Curved Backgrounds . . . 120
15.8 Gates and Empirical Interfaces . . . 120

16 Compact Objects, Horizons, Wormholes, and Causality 122
16.1 Stationary Spherical Configurations and Spectral TOV . . . 122
16.2 Horizons, Surface Gravity, and Entropy Balance . . . 123
16.3 Ultracompact, Horizonless Spectral Solitons . . . 123
16.4 Wormholes: Throats, Energy Conditions, and QEIs . . . 124
16.5 Chronology Protection and Global Hyperbolicity . . . 124
16.6 Diagnostics and Observational Windows . . . 125
16.7 Gates Summary for Strong–Field Sector . . . 125

17 Global Parameter Maps: From Spectral Couplings to Observables 126
17.1 Scope and Canonical Parameter Vector . . . 126
17.2 Gate–Aware Priors and Validity Domains . . . 126
17.3 Relativistic/PN Tests (Solar–System & Binary Pulsars) . . . 127
17.4 Electromagnetism/QED Corrections . . . 127
17.5 Scattering & Forward Positivity . . . 127
17.6 Cosmology: $(n_s, r, f_{\text{NL}}, g_*, \Delta N_{\text{eff}}, w_{\text{eff}})$ . . . 127
17.7 Lensing, Time Delays, and EHT . . . 128
17.8 Gravitational Waves: Propagation and Ringdown . . . 128
17.9 Defects, LSS Morphology, and Maps to Statistics . . . 128
17.10Jacobians, Fisher Information, and Error Propagation . . . 128
17.11Canonical Tables (Quick Reference) . . . 129
17.12Analyst's Recipe . . . 129

IV Synthesis, Limits, and Final Maps 130

18 What We Recovered and Where It Breaks 131
18.1 Classical Physics: Recovery Benchmarks . . . 131
18.2 Quantum Physics: Recovery Benchmarks . . . 131
18.3 Predictive Deviations & Stress Points . . . 132
18.4 Controlled Domains and Where It Breaks . . . 132
18.5 Kill Criteria (Operational) . . . 132

Appendix I: Final Spectral-to-Observable Tables 134
PN & Relativistic Tests . . . 134
Wave Speeds . . . 134
QED Precision . . . 134
Scattering & Positivity . . . 134
CMB/LSS . . . 135

Lensing & GW 135

V Roadmap Beyond Volume III 136

19 Volume IV — Spectral Particle Physics 137
19.1 Emergent Gauge Symmetries from Fiber Isometries 137
19.2 't Hooft Matching and Anomaly Accounting 137
19.3 Masses and Mixing 138
19.4 Effective Strong Dynamics 138

20 Volume V — Spectral Quantum Foundations 139
20.1 Damascus Interpretation: Compact Proof 139
20.2 Entanglement, No-Signalling, and Threads 139
20.3 Spectral Quantum Control 139

21 Volume VI — Spectral Matter Sciences 140
21.1 Spectral Chemistry 140
21.2 Condensed-Matter Phenomena 140
21.3 Prelude to Spectral Biology 140

22 Post-V3 Experimental Program 141
22.1 CMB/LSS 141
22.2 Gravitational Waves 141
22.3 Precision Labs 141
22.4 Table-top/Guided Experiments 141

23 Decisive Discriminants vs. ΛCDM/QFT 142

24 Global Kill Criteria (Post-V3) 143

25 Near-Term Deliverables (During V3⇒V4) 144

Part I

Spectral Quantum Foundations

Chapter 1

Foundations of Spectral Quantum Mechanics: State, Measurement, and Entanglement

1.1 Introduction

This chapter promotes the Quantum–Aether framework from a classical/emergent description (Volumes I–II) to a full quantum theory. The central move is ontological: the quantum *state* is not a free-floating probability amplitude but a real, organized configuration (coherent mode/soliton) of a single spectral medium with fields $\mathcal{S}(x) = \{\Phi, \rho, \eta, \chi^\mu, T\}$. Spacetime geometry, causal cones, and clocks are *derived* properties of this medium via the effective metric $g_{\text{eff}}[\mathcal{S}]$ and spectral derivative D_μ (Vol. I), while early-universe dynamics (Vol. II) fixes viable background classes. Building on these, we construct spectral quantum mechanics along four pillars:

(i) Hilbert structure from the quadratic spectral action. Starting from the second-order expansion of the unified action around an ordered background S_0,

$$\mathcal{L}^{(2)} = \tfrac{1}{2}\,\delta\mathbf{S}^\top \left[\mathsf{K}\,\partial_t^2 + \mathsf{C}\,\partial_t + \mathsf{Z}^{ij}\nabla_i\nabla_j + \mathsf{M}^2\right]\delta\mathbf{S},$$

we extract a covariant symplectic form, a compatible complex structure, and the inner product that defines the one-particle space. Quantization yields canonical algebras and Fock/coherent sectors *without* imposing any background metric by hand. Positivity of the time-kinetic block ($\mathsf{K} \succ 0$) and hyperbolicity of the principal symbol (cones lying inside those of g_{eff}) become the *no-ghost* and *causality* gates for quantum kinematics. In this sense, uncertainty relations emerge as positivity statements of the quadratic form rather than axioms.

(ii) Events and instruments as open spectral couplings. Physical interventions (sources, apertures, detectors) are localized couplings between system operators built from $\Phi, \ldots$ and instrument operators built from the dense/viscous sectors ρ, η and the director χ^μ. Under coarse-graining within the cone of g_{eff}, reduced dynamics takes a *causal Lindblad* (GKSL) form

$$\dot\rho = -i[H_{\text{eff}}, \rho] - \sum_a \gamma_a(\mathcal{S}) \int d^3x \left(L_a \rho L_a^\dagger - \tfrac{1}{2}\{L_a^\dagger L_a, \rho\}\right),$$

with jump operators $L_a[\eta, \rho, \chi^\mu; D_\mu]$ built from the same spectral data. Complete positivity and no-signalling follow from locality of the kernels and the emergent light-cone structure.

A non-negative entropy production rate $\dot{S}_{\mathrm{vN}} \geq 0$ arises whenever $\eta > 0$, tying the *arrow of time* to dissipative channels in the medium.

(iii) Measurement as spectral flooding; Born weights as stationary shares. "Collapse" is replaced by a dynamical process: spectral flooding into stable *pointer modes* of the instrument—eigenmodes stabilized by large ρ and η with phase locking mediated by χ^μ. In the long-time, weakly driven limit, the observed frequencies approach *stationary energy shares* across pointer subspaces, $P_i \to \int |\Psi_i(x)|^2 d^3x$, providing a derivation of Born's rule as an *effective limit* rather than a postulate. Finite-rate and partial-locking regimes predict controlled deviations (bounded by locking and decoherence times) that define concrete experimental targets.

(iv) Entanglement as structural nonseparability. Entangled states are single, extended spectral configurations whose "lobes" are connected by internal χ^μ*-threads*. This yields quantum correlations up to the Tsirelson bound while maintaining relativistic causality: spacelike instruments commute because the GKSL kernels have compact support inside the g_{eff} cone. Local degradation of threads suppresses nonlocal correlations *without* enabling superluminal signalling, offering testable *decohesion pathways* distinct from environmental decoherence alone.

Programmatic payoffs. (1) A unified account of state, measurement, and entanglement grounded in a single medium eliminates the classical/quantum split and the need for external collapse postulates. (2) The same consistency gates that ruled Volumes I–II (no ghosts, causality, positivity, restoration) now constrain open-system data $(H_{\mathrm{eff}}, L_a, \gamma_a)$. (3) Falsifiable signatures arise naturally: phase-only fringe shifts at fixed counts (double slit), *colored Zeno* crossovers tunable by the instrument spectrum, and controlled angular anisotropies inherited from χ^μ in multi-port interferometry. (4) Interfaces to emergent gauge symmetry and to the Standard Model (to be developed in subsequent chapters/volumes) are clarified: fiber isometries generate gauge structure while the η-controlled arrow underwrites operational notions of measurement and amplification.

In sum, the chapter supplies the mathematical and operational backbone of *Spectral Quantum Mechanics*: the Hilbert space is extracted from the quadratic spectral action; measurements are physical, causal, and dissipative processes in the same medium; entanglement is structural rather than metaphysical; and standard quantum statistics appear as stabilized, long-time limits—all under the umbrella of the aether's consistency gates.

1.2 Ontological Postulates and Operational Scope

This section specifies the *beables* of Spectral Quantum Mechanics (SQM) and the level of description at which experiments are modeled. We (i) state what the spectral medium is and how geometric notions emerge from it; (ii) define *events* and *instruments* as localized couplings plus a coarse-graining map consistent with the effective light cone; and (iii) elevate four *quantum gates* (GQ1–GQ4) that every reduced description must satisfy. Throughout, background notions $(g_{\mathrm{eff}}, D_\mu, e^a{}_\mu, \omega_\mu)$ refer to the constructions of Vol. I (Ch. 2–3), and cosmological backgrounds to Vol. II.

1.2.1 The spectral medium and its fields

Ontic carrier. Reality is modeled as a single spectral medium with internal fields

$$\mathcal{S}(x) \;=\; \{\; \Phi(x),\; \rho(x),\; \eta(x),\; \chi^\mu(x),\; T(x) \;\}, \qquad x \in \mathcal{M}, \tag{1.1}$$

where $\mathcal{M}$ is a smooth 4-manifold without a *prior* metric. The fields encode, respectively, spectral phase/coherence Φ, density ρ, viscosity/dissipation $\eta \geq 0$, internal director χ^μ (breaking/isotropizing directionality), and an internal temporal order parameter T.

Emergent geometry and spectral derivative. A coframe $e^a{}_\mu[\mathcal{S}]$ and a torsionful Palatini-type connection $\omega_\mu[\mathcal{S}]$ are *derived* from $\mathcal{S}$ (Vol. I):

$$g^{\text{eff}}_{\mu\nu}(\mathcal{S}) \;=\; \eta_{ab}\, e^a{}_\mu e^b{}_\nu, \qquad D_\mu \;\equiv\; \partial_\mu + \mathcal{A}_\mu[\mathcal{S}], \qquad \mathcal{A}_\mu \in \mathfrak{iso}(F)\,, \tag{1.2}$$

with F the internal spectral fiber. All kinetic and interaction terms are built from D_μ and curvature tensors of (e, ω); no background metric is allowed in the Lagrangian.

Kinematic regimes. Three asymptotic regimes organize the effective theory:

1. **Pre-geometric (microscopic).** Strong gradients and/or large η; g_{eff} is undefined or highly fluctuating. Dynamics is governed by local spectral couplings and short causal neighborhoods.

2. **Effective geometric.** Coarse-graining over scales $\ell_{\text{cg}} \gg \xi$ (correlation length) yields a smooth g_{eff}, with principal symbols defining cones inside which front velocities propagate. Quantization of linearized modes is meaningful (this is the working regime of SQM).

3. **Hydrodynamic.** Long-wavelength, low-frequency sector where constitutive relations close (u^μ, $\sigma_{\mu\nu}$, etc.) and dissipation dominates ($\eta > 0$); entropy production provides an arrow of time.

A small parameter $\varepsilon \equiv \xi/\ell_{\text{cg}} \ll 1$ controls the accuracy of emergent geometric/quantum descriptions.

1.2.2 Events, instruments, and coarse description

Events. An *event* is a localized variation of $\mathcal{S}$ driven by a compact kernel $g(x)$ acting during a finite window:

$$\delta\mathcal{S}(x) \;=\; \int_{\mathcal{O}} d^4y\; g(x-y)\, \mathcal{J}[\mathcal{S}](y), \qquad \operatorname{supp} g \subset \mathcal{N}_\Delta \subset \mathcal{C}^+_{g_{\text{eff}}}, \tag{1.3}$$

where $\mathcal{O}$ is a bounded spacetime region, $\mathcal{N}_\Delta$ a neighborhood of size $\Delta = (\ell_{\text{loc}}, \tau_{\text{loc}})$, and $\mathcal{C}^+_{g_{\text{eff}}}$ the future light cone of g_{eff} at the relevant background. The source functional $\mathcal{J}$ is built from admissible spectral operators (phase twists, density kicks, director torques).

Instruments. An *instrument* is a controlled subsystem with degrees of freedom

$$\mathcal{I}(x) = \{\rho_{\text{inst}}(x),\; \eta_{\text{inst}}(x),\; \Pi_i(x)\}, \tag{1.4}$$

where $\rho_{\text{inst}}, \eta_{\text{inst}}$ provide high density/viscosity (amplification & dissipation) and Π_i are *pointer fields* (metastable modes). The coupling to a *system* sector $\mathcal{S}_{\text{sys}}$ takes the bilinear form

$$\mathcal{L}_{\text{coup}} = \sum_a \lambda_a \int d^4x\, K_a(x)\, \mathcal{O}_a[\mathcal{S}_{\text{sys}}](x)\, \mathcal{P}_a[\mathcal{I}](x), \tag{1.5}$$

with kernels K_a supported inside the g_{eff}-cone (microcausality).

Coarse graining and reduced dynamics. Let $\mathcal{E}_\Delta$ be a CPTP coarse-graining map over a spacetime cell of size Δ within the causal cone:

$$\varrho_{\rm sys}(t+\delta t) \;=\; \mathcal{E}_\Delta\Big[\varrho_{\rm tot}(t)\Big] \;\equiv\; {\rm Tr}_{\mathcal{I}}\Big[\mathcal{U}_\Delta\,\varrho_{\rm tot}(t)\,\mathcal{U}_\Delta^\dagger\Big]\,, \qquad \mathcal{U}_\Delta = {\rm T}\exp\!\int_t^{t+\delta t} dt'\,H_{\rm int}(t'), \tag{1.6}$$

where ${\rm Tr}_{\mathcal{I}}$ traces instrument/environment and $H_{\rm int}$ is generated by (1.5). In the Markovian limit (finite memory $\ll \delta t$) this yields a GKSL generator $\mathcal{L}$ (Sec. **??**):

$$\dot\varrho_{\rm sys} \;=\; \mathcal{L}[\varrho_{\rm sys}] = -i[H_{\rm eff},\varrho_{\rm sys}] + \sum_\alpha \gamma_\alpha \int d^3x\,\Big(L_\alpha \varrho_{\rm sys} L_\alpha^\dagger - \tfrac{1}{2}\{L_\alpha^\dagger L_\alpha,\varrho_{\rm sys}\}\Big), \quad \gamma_\alpha \geq 0, \tag{1.7}$$

with $L_\alpha = L_\alpha[\rho_{\rm inst},\eta_{\rm inst},\chi^\mu; D_\mu]$ local inside $\mathcal{C}^+_{g_{\rm eff}}$.

Locality window. Causality at the reduced level demands a *locality window*

$$\operatorname{supp} K_a \subset \mathcal{C}^+_{g_{\rm eff}}(x_0), \qquad \tau_{\rm mem} \ll \delta t \ll \tau_{\rm drift}, \qquad \ell_{\rm mem} \ll \ell_{\rm cg} \ll \ell_{\rm drift}, \tag{1.8}$$

with $\tau_{\rm mem}$, $\ell_{\rm mem}$ the instrument's memory scales and $\tau_{\rm drift}$, $\ell_{\rm drift}$ the slow background-drift scales. Violation of (1.8) signals the breakdown of the Markovian GKSL limit and triggers nonlocal kernels still confined within $\mathcal{C}^+_{g_{\rm eff}}$.

1.2.3 Foundational gates for the quantum sector

We collect the *quantum gates* that any admissible reduced model must satisfy. They are the quantum counterparts of the classical gates (Vol. I) and will be enforced in all constructions below.

GQ1 — Causality (front velocities). For every Heisenberg operator $\mathcal{O}(x)$ of the system sector and any spacelike-separated $\mathcal{O}'(y)$ w.r.t. $g_{\rm eff}$,

$$[\mathcal{O}(x),\mathcal{O}'(y)] \;=\; 0 \quad \text{if} \quad (x-y) \notin \overline{\mathcal{C}^{\pm}_{g_{\rm eff}}}. \tag{1.9}$$

Equivalently, the *front velocity* of the retarded Green function of the quadratic sector obeys $v_{\rm front} \leq c_{\rm eff}$, the null speed of $g_{\rm eff}$.

GQ2 — Complete positivity (CP) and divisibility. The reduced time evolution $\{\mathcal{E}_{t_2,t_1}\}_{t_2\geq t_1}$ is CPTP and CP-divisible on the locality window:

$$\mathcal{E}_{t_3,t_1} = \mathcal{E}_{t_3,t_2}\circ\mathcal{E}_{t_2,t_1}, \qquad \mathcal{E}_{t_2,t_1} = \exp\Big[(t_2-t_1)\mathcal{L}_{t_1}\Big], \quad \mathcal{L}_{t_1} \text{ GKSL with } \gamma_\alpha \geq 0. \tag{1.10}$$

Microscopically this rests on (i) positivity of the Kossakowski matrix from instrument correlators and (ii) locality of kernels inside $\mathcal{C}^+_{g_{\rm eff}}$.

GQ3 — Entropy production (arrow of time). For $\eta_{\rm inst} > 0$ the von Neumann entropy satisfies Spohn's inequality with respect to a steady state ϱ_* of $\mathcal{L}$,

$$\sigma_{\rm diss} \;\equiv\; -\frac{d}{dt}\,S(\varrho_{\rm sys}\|\varrho_*) \;=\; -{\rm Tr}\Big[\mathcal{L}\varrho_{\rm sys}(\ln\varrho_{\rm sys} - \ln\varrho_*)\Big] \;\geq 0, \tag{1.11}$$

and reduces, for quadratic sectors, to the spectral-production form of Vol. I:

$$\partial_t \delta\langle\Phi| * |\Phi\rangle \equiv \int (\delta\Phi) g(\nabla,\mu)\Phi d\Omega \tag{1.12}$$

GQ4 — Restoration (linearity/isotropy limits). In the limits of (i) vanishing dissipation and anisotropy,

$$\eta \downarrow 0, \quad \chi^\mu \text{ fluctuations } \downarrow 0, \quad \ell_{\rm cg}/\ell_{\rm drift} \downarrow 0, \tag{1.13}$$

the reduced dynamics restores unitary linear evolution with isotropic principal symbol:

$$\dot{\varrho}_{\rm sys} \to -i[H^{(\rm iso)}_{\rm eff}, \varrho_{\rm sys}], \qquad c_{s,\parallel} \to c_{s,\perp}, \quad c_T \to 1, \tag{1.14}$$

and standard quantum-mechanical kinematics in the $g_{\rm eff}$ frame. Conversely, controlled deviations at small but finite $(\eta, \Delta c)$ define the space of *falsifiable* signatures (Vol. II, Ch. 4).

Together, (1.9)–(1.14) delimit the operational scope of SQM: quantization proceeds from the quadratic spectral action (Hilbert structure); interventions are localized, causal couplings to instrument sectors; reduced dynamics is CP and entropy-producing when $\eta > 0$; and ordinary quantum theory is recovered as a restoration limit of the same medium.

1.3 From the Quadratic Spectral Action to a Hilbert Space

We derive the kinematic quantum structure from the quadratic (linearized) spectral action about an ordered background S_0 (Vol. I, Ch. 2–3; Vol. II, Sec. **??**). The construction follows the standard symplectic→Kähler→Hilbert pipeline: (i) normal-mode decomposition of the quadratic Lagrangian; (ii) canonical phase space and symplectic form on a Cauchy slice Σ of $g_{\rm eff}[S_0]$; (iii) selection of a complex structure J compatible with the aether time-flow u^μ; (iv) one-particle pre-Hilbert space and Fock (or coherent/solitonic) representations; (v) uncertainty relations as positivity of the quadratic form. Dissipative/anisotropic corrections are kept symbolic but the Fock construction itself uses the conservative window (GQ4).

1.3.1 Quadratic action and mode decomposition

Let $\delta\mathbf{S}$ collect the linear perturbations of the spectral fields about S_0 in a fixed background gauge (e.g. comoving-aether for scalars, TT for tensors). Up to boundary terms one can always rewrite the quadratic Lagrangian density in a first-order-in-time form

$$\mathcal{L}^{(2)} = \frac{1}{2}\,(\partial_t\delta\mathbf{S})^\top \mathsf{K}\,(\partial_t\delta\mathbf{S}) + \frac{1}{2}\,\delta\mathbf{S}^\top \mathsf{C}_{\rm S}\,(\partial_t\delta\mathbf{S}) - \frac{1}{2}\,\delta\mathbf{S}^\top\Big(\mathsf{Z}^{ij}\nabla_i\nabla_j + \mathsf{M}^2\Big)\delta\mathbf{S}, \tag{1.15}$$

where $\mathsf{K} = \mathsf{K}^\top \succ 0$ is the time-kinetic block (no-ghost gate), $\mathsf{C}_{\rm S} = \frac{1}{2}(\mathsf{C} + \mathsf{C}^\top)$ encodes conservative mixing, while any anti-symmetric/anti-Hermitian dissipative part is treated separately in the reduced (GKSL) dynamics (Sec. 1.2.2, GQ2–GQ3). Spatial gradients appear through the principal block

$$\mathsf{Z}^{ij}k_ik_j = \begin{pmatrix} Z_{\rm sc,\parallel}\,k_\parallel^2 + Z_{\rm sc,\perp}\,k_\perp^2 & * \\ * & Z_{\rm tr,\parallel}\,k_\parallel^2 + Z_{\rm tr,\perp}\,k_\perp^2 \end{pmatrix}, \qquad k_\parallel \equiv \mathbf{k}\cdot\hat{\chi}^{(0)}, \;\; k_\perp^2 = k^2 - k_\parallel^2, \tag{1.16}$$

exhibiting longitudinal/transverse anisotropy w.r.t. the background director $\hat{\chi}^{(0)}$ (Vol. II, Sec. **??**). The Euler–Lagrange equations read

$$\mathsf{K}\,\partial_t^2\delta\mathbf{S} + \mathsf{C}\,\partial_t\delta\mathbf{S} + \Big(\mathsf{Z}^{ij}\nabla_i\nabla_j + \mathsf{M}^2\Big)\delta\mathbf{S} = 0, \qquad \mathsf{C} \equiv \mathsf{C}_{\mathrm{S}} + \mathsf{C}_{\mathrm{A}}, \tag{1.17}$$

with $\mathsf{C}_{\mathrm{A}}^{\top} = -\mathsf{C}_{\mathrm{A}}$ the (possible) antisymmetric piece.

Principal symbol and dispersion. Looking for plane waves $\delta\mathbf{S} \propto e^{i\mathbf{k}\cdot\mathbf{x}-i\omega t}$ on a locally homogeneous patch of Σ,

$$\det\Big[-\omega^2\mathsf{K} - i\omega\,\mathsf{C} + \mathsf{Z}^{ij}k_ik_j + \mathsf{M}^2\Big] = 0. \tag{1.18}$$

In the conservative window ($\mathsf{C}_{\mathrm{A}} = 0$), each propagating eigenmode A obeys

$$\omega_A^2(\mathbf{k}) = c_{A,\parallel}^2\,k_\parallel^2 + c_{A,\perp}^2\,k_\perp^2 + m_{A,\mathrm{eff}}^2 + \cdots, \quad 0 < c_{A,\perp}^2 \le c_{A,\parallel}^2 \le 1, \tag{1.19}$$

ensuring hyperbolicity and sub-luminality inside the g_{eff} cone (GQ1).

Normal modes and orthogonality. Let $\mathbf{e}_A(\mathbf{k},x)$ be eigenvectors of the generalized Helmholtz operator $\mathcal{H} \equiv \mathsf{K}^{-1}\big(\mathsf{Z}^{ij}\nabla_i\nabla_j + \mathsf{M}^2\big)$ on the Cauchy slice Σ (with appropriate boundary conditions). The K-weighted orthonormality reads

$$\int_\Sigma d^3x\,\sqrt{\gamma_{\mathrm{eff}}}\;\mathbf{e}_A^{\top}(\mathbf{k},x)\,\mathsf{K}\,\mathbf{e}_B(\mathbf{k}',x) = \delta_{AB}\,(2\pi)^3\delta^{(3)}(\mathbf{k}-\mathbf{k}'), \tag{1.20}$$

with γ_{eff} the spatial metric determinant induced by g_{eff}. The field admits the mode expansion

$$\delta\mathbf{S}(t,x) = \sum_A \int \frac{d^3k}{(2\pi)^3}\Big[\,Q_A(\mathbf{k},t)\,\mathbf{e}_A(\mathbf{k},x) \;+\; Q_A^*(\mathbf{k},t)\,\mathbf{e}_A^*(\mathbf{k},x)\Big], \tag{1.21}$$

with Q_A obeying $\ddot{Q}_A + \omega_A^2 Q_A = 0$ (to leading order).

1.3.2 Symplectic form, inner product, and canonical variables

Canonical momenta and phase space. From (1.15) the canonical momenta are

$$\delta\mathbf{\Pi} \;\equiv\; \frac{\partial\mathcal{L}^{(2)}}{\partial(\partial_t\delta\mathbf{S})} = \mathsf{K}\,\partial_t\delta\mathbf{S} + \tfrac{1}{2}\,\mathsf{C}_{\mathrm{S}}^{\top}\,\delta\mathbf{S}. \tag{1.22}$$

The classical phase space $\mathcal{P}$ consists of pairs $X = (\delta\mathbf{S}, \delta\mathbf{\Pi})$ on Σ with suitable falloffs.

Symplectic form. The equal-time covariant symplectic form is

$$\Omega\Big(X_1, X_2\Big) = \int_\Sigma d^3x\,\sqrt{\gamma_{\mathrm{eff}}}\Big(\delta\mathbf{S}_1^{\top}\delta\mathbf{\Pi}_2 - \delta\mathbf{S}_2^{\top}\delta\mathbf{\Pi}_1\Big), \tag{1.23}$$

which is conserved under the linear equations of motion. In terms of modes, $\Omega\Big(X_1, X_2\Big) = \sum_A\!\int d^3k\,[\,Q_{1A}^*\dot{Q}_{2A} - Q_{2A}^*\dot{Q}_{1A}\,]$.

Energy quadratic form and positivity gate. The conserved energy functional associated with the aether time-flow u^μ is

$$E^{(2)}[X] = \frac{1}{2}\int_\Sigma d^3x\,\sqrt{\gamma_{\rm eff}}\Big[(\partial_t\delta\mathbf{S})^\top \mathsf{K}(\partial_t\delta\mathbf{S}) + \nabla_i\delta\mathbf{S}^\top \mathsf{Z}^{ij}\nabla_j\delta\mathbf{S} + \delta\mathbf{S}^\top\mathsf{M}^2\delta\mathbf{S}\Big], \tag{1.24}$$

positive-definite when $\mathsf{K}\succ 0$ and the spatial operator is elliptic on Σ (stability gate).

Complex structure and inner product. A complex structure is a real linear map $\mathsf{J}:\mathcal{P}\to\mathcal{P}$ with $\mathsf{J}^2=-\mathbb{1}$, $\Omega(\mathsf{J}X,\mathsf{J}Y)=\Omega(X,Y)$ and such that the bilinear form

$$g(X,Y) \;\equiv\; \Omega(X,\mathsf{J}Y) \tag{1.25}$$

is positive-definite. The triple (Ω,J,g) defines a Kähler structure. Choosing the aether time t as reference, the natural J diagonalizes the harmonic oscillators Q_A by splitting positive/negative frequencies,

$$\mathsf{J}\;:\;(Q_A,\dot Q_A)\mapsto\Big(-\omega_A^{-1}\dot Q_A,\;\omega_A\,Q_A\Big),\qquad g=\sum_A\int d^3k\,\Big(|\dot Q_A|^2+\omega_A^2|Q_A|^2\Big)/2, \tag{1.26}$$

which is equivalent (on-shell) to $E^{(2)}$. The one-particle pre-Hilbert space $\mathcal{H}_1$ is the completion of the complexified phase space under the inner product

$$\langle X,Y\rangle \;\equiv\; \tfrac{1}{2}\Big[g(X,Y)+i\,\Omega(X,Y)\Big] = \tfrac{1}{2}\,\Omega\Big(X,\mathsf{J}Y\Big)+\tfrac{i}{2}\,\Omega(X,Y). \tag{1.27}$$

1.3.3 Quantization and equal–time algebra

Canonical commutation relations (CCR). Promote $(\delta\mathbf{S},\delta\mathbf{\Pi})$ to operator-valued distributions on Σ with

$$\begin{aligned}\Big[\widehat{\delta S}^A(t,\mathbf{x}),\,\widehat{\delta S}^B(t,\mathbf{y})\Big]&=0,\qquad \Big[\widehat\Pi_A(t,\mathbf{x}),\,\widehat\Pi_B(t,\mathbf{y})\Big]=0,\\ \Big[\widehat{\delta S}^A(t,\mathbf{x}),\,\widehat\Pi_B(t,\mathbf{y})\Big]&= i\,\delta^A{}_B\,\frac{\delta^{(3)}(\mathbf{x}-\mathbf{y})}{\sqrt{\gamma_{\rm eff}(t,\mathbf{x})}}\,,\end{aligned} \tag{1.28}$$

where A,B run over the field components in $\delta\mathbf{S}$. (For linearized *fermionic* sectors, one analogously builds a CAR algebra from the spectral Dirac operator $\not{D}[\mathcal{S}]$.)

Mode operators and Fock space. With the K-orthonormal basis (1.20), define

$$\begin{aligned}\widehat a_A(\mathbf{k}) &= \sqrt{\frac{\omega_A}{2}}\,\widehat Q_A(\mathbf{k})+\frac{i}{\sqrt{2\omega_A}}\,\widehat{\dot Q}_A(\mathbf{k}),\qquad \Big[\widehat a_A(\mathbf{k}),\widehat a_B^\dagger(\mathbf{k}')\Big]=(2\pi)^3\delta_{AB}\,\delta^{(3)}(\mathbf{k}-\mathbf{k}'),\\ \widehat{\delta\mathbf{S}}(t,x) &= \sum_A\int\frac{d^3k}{(2\pi)^3}\frac{1}{\sqrt{2\omega_A}}\Big[\widehat a_A(\mathbf{k})\,e^{-i\omega_A t}\,\mathbf{e}_A(\mathbf{k},x)+\text{h.c.}\Big].\end{aligned} \tag{1.29}$$

The vacuum $|0\rangle$ is the J-ground state ($\widehat a_A|0\rangle=0$), and the Fock space is $\mathcal{F}=\bigoplus_{n=0}^\infty \mathcal{H}_1^{\otimes_s n}$.

Coherent/solitonic sectors. Stable nonlinear coherent configurations (solitons, defects) define inequivalent sectors as displaced vacua:

$$|\alpha;\text{sol}\rangle = \exp\Big(\sum_A\int d^3k\,[\alpha_A(\mathbf{k})\,\widehat a_A^\dagger-\text{h.c.}]\Big)\,|0\rangle, \tag{1.30}$$

or, more generally, as quantization on a nontrivial classical background $S_{\rm sol}$ (Vol. I, Ch. 4; Vol. II, Ch. **??**). Particle-like excitations are then small oscillations around $S_{\rm sol}$ rather than S_0.

Open-system window. Small anti-Hermitian pieces (e.g. Rayleigh terms $\propto \eta$) are not quantized directly; instead they induce GKSL evolution for $\widehat{\varrho}_{\rm sys}$ with local Lindblad operators $L_\alpha[\mathcal{S}]$ (Sec. 1.2.2, GQ2–GQ3). The CCR structure (1.28) remains intact while expectation values satisfy entropy production inequalities.

1.3.4 Uncertainty as positivity of the quadratic form

Let $\widehat{X}[\mathbf{f}]$ be a smeared linear observable of the form

$$\widehat{X}[\mathbf{f}] = \int_\Sigma d^3x\,\sqrt{\gamma_{\rm eff}}\,\Big(\mathbf{f}_S^\top\,\widehat{\delta\mathbf{S}} + \mathbf{f}_\Pi^\top\,\widehat{\mathbf{\Pi}}\Big), \qquad \mathbf{f} = (\mathbf{f}_S, \mathbf{f}_\Pi). \tag{1.31}$$

From the CCR one finds $[\widehat{X}[\mathbf{f}], \widehat{X}[\mathbf{g}]] = i\,\Omega(\mathbf{f},\mathbf{g})$ with Ω as in (1.23). For any state $\widehat{\varrho}$ the covariance matrix $\Sigma_{\mathbf{fg}} = \frac{1}{2}\langle\{\Delta\widehat{X}[\mathbf{f}], \Delta\widehat{X}[\mathbf{g}]\}\rangle$ satisfies the Robertson inequality

$$\Sigma_{\mathbf{ff}}\,\Sigma_{\mathbf{gg}} - \Big|\Sigma_{\mathbf{fg}}\Big|^2 \;\geq\; \frac{1}{4}\Big|\Omega(\mathbf{f},\mathbf{g})\Big|^2. \tag{1.32}$$

Specializing to canonical pairs $(\mathbf{f}_S, \mathbf{f}_\Pi) = (\delta^A{}_B\,\delta^{(3)}, 0)$ and $(0, \delta_A{}^B\,\delta^{(3)})$ yields generalized Heisenberg bounds with the $\sqrt{\gamma_{\rm eff}}$ weight. In the mode basis one has

$$\Delta Q_A(\mathbf{k})\,\Delta P_A(\mathbf{k}) \;\geq\; \frac{1}{2}, \qquad P_A \equiv \dot{Q}_A, \tag{1.33}$$

and the vacuum saturates them: $\Delta Q_A^2 = 1/(2\omega_A)$, $\Delta P_A^2 = \omega_A/2$. Equivalently, (1.33) follows from positivity of the quadratic energy form (1.24) together with Cauchy–Schwarz in the Kähler metric g:

$$\Big|\,\Omega(X,Y)\,\Big|^2 \;\leq\; 4\,E^{(2)}[X]\;E^{(2)}[Y]. \tag{1.34}$$

Thus uncertainty is not a separate postulate: it is a direct corollary of (i) the symplectic structure fixed by the quadratic action and (ii) positivity of the kinetic and gradient blocks (GQ1, GQ4).

Summary. The quadratic spectral action canonically induces a conserved symplectic form, a preferred complex structure tied to the aether time, and a positive energy metric; together they define the one-particle Hilbert space and its Fock/coherent extensions. Anisotropy relative to $\hat{\chi}^{(0)}$ enters only through the dispersion data $(c_{A,\parallel}, c_{A,\perp})$ and the spatial operator Z^{ij}, leaving the CCR and uncertainty structure universal.

1.4 Events as Couplings and the Spectral Lindblad Generator

We formalize *events* (preparations, probes, "measurements") as localized couplings between a *system* subspace of the spectral fields and an *instrument* (apparatus/ambient) subspace, both embedded in the same Quantum–Aether medium. Starting from a microscopic interaction Lagrangian with compact support inside the $g_{\rm eff}$ cone, we perform a controlled coarse–graining (Born–Markov–secular where appropriate) to obtain a causal, completely positive (CP) Lindblad generator. Entropy production and the arrow of time are traced to $\eta > 0$ (open channels), in agreement with the gates GQ1–GQ4.

1.4.1 Microscopic interaction Lagrangians

We partition the degrees of freedom into a *system* sector (e.g. a subset of modes of Φ or a localized solitonic collective coordinate) and an *instrument* sector (high–ρ, high–η fields, pointer variables). A general local interaction density reads

$$\mathcal{L}_{\rm int}(x) = \sum_a g_a(x)\, \mathcal{O}^{(a)}_{\rm sys}\Big[\Phi, \delta\mathbf{S}_{\rm sys}; D_\mu\Big]\, \mathcal{A}^{(a)}_{\rm app}\Big[\rho, \eta, \chi^\mu, \delta\mathbf{S}_{\rm app}; D_\mu\Big], \tag{1.35}$$

where $g_a(x)$ are smooth, compactly supported profiles (switching and spatial window), and D_μ is the spectral derivative (Vol. I, Ch. 2). Directional response is controlled by the background director χ^μ through contractions $D_\parallel \equiv \chi^\mu D_\mu$ and $D_\perp$.

Typical couplings.

$$\begin{aligned}
&\text{(Amplitude/“position”):}\quad \mathcal{O}_{\rm sys} = \Phi, \qquad \mathcal{A}_{\rm app} = Q_{\rm pointer},\\
&\text{(Current/“momentum”):}\quad \mathcal{O}_{\rm sys} = J^\mu[\Phi] \equiv \Phi^\dagger \overleftrightarrow{D}^\mu \Phi, \qquad \mathcal{A}_{\rm app} = A_\mu[\rho,\eta,\chi],\\
&\text{(Dephasing):}\quad \mathcal{O}_{\rm sys} = \Phi^\dagger\Phi, \qquad \mathcal{A}_{\rm app} = \Xi[\eta]\ \ \text{(viscous noise).}
\end{aligned} \tag{1.36}$$

All choices respect the locality window enforced by $g_{\rm eff}$ (GQ1) and are polynomial in fields/derivatives at the scale of interest.

1.4.2 Markovian coarse–graining and locality

Let $\widehat{\varrho}_{\rm tot}$ evolve under $H = H_{\rm sys} + H_{\rm app} + H_{\rm int}$ with $H_{\rm int} = -\int d^3x\, \mathcal{L}_{\rm int}$. We assume an initial factorized state,

$$\widehat{\varrho}_{\rm tot}(t_0) = \widehat{\rho}(t_0) \otimes \widehat{\rho}^{\rm ref}_{\rm app}, \qquad \widehat{\rho}^{\rm ref}_{\rm app} \text{ stationary (e.g. KMS) for } H_{\rm app}. \tag{1.37}$$

In the interaction picture and to second order (Born), the Nakajima–Zwanzig equation gives

$$\dot\rho(t) = -\int_{t_0}^{t} ds\ \mathrm{Tr}_{\rm app}\Big[H_{\rm int}(t), \Big[H_{\rm int}(s), \rho(t)\otimes\rho^{\rm ref}_{\rm app}\Big]\Big] \ +\ \mathcal{O}(g^3), \tag{1.38}$$

where $\rho(t) \equiv \mathrm{Tr}_{\rm app}\, \varrho_{\rm tot}(t)$. Denote apparatus two–point correlators

$$\mathcal{C}_{ab}(x,y) \equiv \mathrm{Tr}_{\rm app}\Big\{\mathcal{A}^{(a)}_{\rm app}(x)\, \mathcal{A}^{(b)}_{\rm app}(y)\, \rho^{\rm ref}_{\rm app}\Big\}. \tag{1.39}$$

Causality/locality. Microcausality in the emergent metric implies $\mathcal{C}_{ab}(x,y) = 0$ for spacelike separation w.r.t. $g_{\rm eff}$, while finite signal speed and viscous damping ($\eta > 0$) yield a finite correlation time τ_c and correlation length ℓ_c.

Markov window. If the system evolves on scales $T_{\rm sys} \gg \tau_c$ and $L_{\rm sys} \gg \ell_c$, one may approximate memory kernels as local (time-convolutionless, TCL) and adopt a secular average over fast oscillations (when a discrete spectrum is present). The result is a generator local in time and (to leading order) in space within the $g_{\rm eff}$ cone.

Rates from spectral densities. Fourier transforming along the aether time t,

$$\widetilde{\mathcal{C}}_{ab}(\omega;\hat{\mathbf{k}}) = \int d\tau\, e^{i\omega\tau} \int d^3\xi\, e^{-i\mathbf{k}\cdot\boldsymbol{\xi}}\, \mathcal{C}_{ab}(t,\mathbf{x};t-\tau,\mathbf{x}-\boldsymbol{\xi}), \qquad \hat{\mathbf{k}} = \mathbf{k}/|\mathbf{k}|, \tag{1.40}$$

defines positive semidefinite Kossakowski matrices by Bochner's theorem. Directional dependence enters through $\hat\chi^{(0)}\cdot\hat{\mathbf{k}}$. Markovian rates are

$$\gamma_{ab}(\omega,\hat{\mathbf{k}}) \ =\ g_a g_b\, \widetilde{\mathcal{C}}_{ab}(\omega;\hat{\mathbf{k}})\,, \tag{1.41}$$

evaluated at Bohr frequencies of $H_{\rm sys}$.

1.4.3 Causal, CP Lindblad form from spectral data

Under the Born–Markov–secular approximations and within the g_{eff} light cone, the reduced state ρ obeys a Gorini–Kossakowski–Sudarshan–Lindblad (GKSL) equation

$$\dot{\rho} = -i\big[H_{\text{eff}} + H_{\text{LS}}, \rho\big] + \sum_{\alpha,\beta}\int d^3x \left(\mathsf{K}_{\alpha\beta}\, L_\alpha(x)\, \rho\, L_\beta^\dagger(x) - \tfrac{1}{2}\,\mathsf{K}_{\alpha\beta}\,\{L_\beta^\dagger(x)L_\alpha(x), \rho\}\right), \quad (1.42)$$

where:

- H_{eff} is the system Hamiltonian inherited from H_{sys} (quadratic spectral action or its few–mode reduction);
- H_{LS} is the Lamb-shift Hamiltonian from principal values of (1.39);
- $L_\alpha(x) \equiv L_\alpha[\eta, \rho, \chi^\mu; D_\mu](x)$ are *local Lindblad densities* built from system observables (e.g. $\Phi,\ J^\mu,\ \Phi^\dagger\Phi,\ D_\parallel\Phi$);
- $\mathsf{K}_{\alpha\beta}$ (the Kossakowski matrix) is positive semidefinite, $\mathsf{K} \succeq 0$, with entries given by angle– and frequency–integrated rates (1.41).

Complete positivity. Since $\mathsf{K} \succeq 0$, (1.42) generates a CP, trace–preserving semigroup. The CP property follows from the PSD nature of environment correlators $\widetilde{\mathcal{C}}_{ab}$ (Bochner) and the secular block–diagonalization in Bohr sectors.

Causality/no–signalling. Locality is enforced by (i) the δ-like support of kernels at scales $\gg (\tau_c, \ell_c)$ and (ii) vanishing of commutators $[L_\alpha(x), L_\beta(y)] = 0$ for spacelike (x, y) in g_{eff}. Thus signals cannot propagate outside the g_{eff} cone (GQ1).

Representative Lindblad structures.

$$\begin{aligned}
&\text{(Pure dephasing)} \quad L(x) = \sqrt{\gamma_\phi(x)}\,\Phi^\dagger\Phi(x),\\
&\text{(Amplitude damping)} \quad L(x) = \sqrt{\gamma_-(x)}\,\Phi(x), \qquad L^\dagger(x) = \sqrt{\gamma_+(x)}\,\Phi^\dagger(x),\\
&\text{(Direction–selective diffusion)} \quad L_\parallel(x) = \sqrt{\gamma_\parallel(x)}\,D_\parallel\Phi(x), \quad L_\perp(x) = \sqrt{\gamma_\perp(x)}\,D_\perp\Phi(x),
\end{aligned} \quad (1.43)$$

with $\gamma_\parallel \neq \gamma_\perp$ encoding anisotropy via $\hat{\chi}^{(0)}$.

1.4.4 Entropy production and arrow of time

Let $S_{\text{vN}}(\rho) \equiv -\operatorname{Tr}\rho\ln\rho$. For a GKSL evolution (1.42) one has Spohn's inequality relative to any stationary state ρ_* of the semigroup:

$$\sigma[\rho] \;\equiv\; -\frac{d}{dt}\,S\big(\rho\big\|\rho_*\big) = -\frac{d}{dt}\operatorname{Tr}\big(\rho\ln\rho - \rho\ln\rho_*\big) \;\geq\; 0, \quad (1.44)$$

with equality at $\rho = \rho_*$. If the apparatus is KMS at temperature T (local equilibrium), (1.44) yields the Clausius form

$$\dot{S}_{\text{vN}}(\rho) \;-\; \frac{\dot{Q}}{T} \;\geq\; 0, \qquad \dot{Q} \equiv \operatorname{Tr}\big(H_{\text{eff}}\,\dot{\rho}\big), \quad (1.45)$$

so entropy production is non–negative.

Dephasing–dominated (unital) limit. For dephasing/diffusive channels (typical when $\eta > 0$ dominates and detailed balance is nearly unital), the dissipator is unital, $\sum_\alpha L_\alpha L_\alpha^\dagger = \sum_\alpha L_\alpha^\dagger L_\alpha$, and

$$\dot{S}_{\mathrm{vN}}(\rho) = \sum_\alpha \gamma_\alpha \,\mathrm{Tr}\Big(L_\alpha^\dagger L_\alpha\,\rho - L_\alpha \rho L_\alpha^\dagger\Big) \;\geq\; 0, \tag{1.46}$$

with equality iff $[L_\alpha, \rho] = 0$ for all α. Thus, $\eta > 0$ *opens dissipative channels that monotonically increase von Neumann entropy*, selecting the time arrow (cf. Vol. II, Sec. **??**). In the Hamiltonian limit $\eta \downarrow 0$ the generator becomes unitary and $\dot{S}_{\mathrm{vN}} \to 0$.

Summary. Local event couplings (1.35) and finite spectral correlation scales (τ_c, ℓ_c) imply a causal, Lindblad–type reduced dynamics (1.42), with CP guaranteed by positivity of apparatus correlators and no–signalling enforced by the g_{eff} cone. Entropy production is non–negative by Spohn, and in the dephasing–dominated regime (1.46) renders the arrow of time explicit, tying it to $\eta > 0$ in the spectral medium.

1.5 Measurement as Spectral Flooding and the Born Limit

We model a measurement as an *open, direction–aware energy transfer* from a selected system observable to macroscopic pointer sectors of the same spectral medium. The key dynamical ingredient is the *spectral flooding* induced by viscous/instrumental channels ($\eta > 0$) which drives phase locking into stable instrument eigenmodes. In this section we: (i) define pointer sectors and locking rates; (ii) derive the Born weights as the *stationary energy–share* of the flooding process under a symmetry hypothesis on channels; (iii) bound deviations at finite rates; (iv) characterize Zeno/anti–Zeno regimes with a colored Zeno parameter.

1.5.1 Pointer sectors and phase locking

Let the instrument be tuned to an observable (spectral functional) $\widehat{\mathcal{A}}$ on the system sector, with discrete spectral resolution $\{\Pi_i\}$, $\Pi_i\Pi_j = \delta_{ij}\Pi_i$, $\sum_i \Pi_i = \mathbb{1}$. We say the instrument admits *pointer sectors* if, for some local Lindblad densities $L_i(x)$ (Sec. 1.4),

$$L_i(x) \;\propto\; \sqrt{\gamma_i(x)}\,\Pi_i\,, \qquad \gamma_i(x) \geq 0, \quad \text{with} \quad \mathrm{supp}\,\gamma_i \subset \mathrm{window}(g_i) \subset g_{\mathrm{eff}}\ \text{cone}. \tag{1.47}$$

The reduced state then evolves (suppressing Lamb shifts)

$$\dot{\rho} \;=\; -i[H_{\mathrm{eff}}, \rho] + \sum_i \int d^3x\, \gamma_i(x) \Big(\Pi_i \rho \Pi_i - \tfrac{1}{2}\{\Pi_i, \rho\}\Big) \;+\; \cdots\,, \tag{1.48}$$

where "$\cdots$" includes instrument diffusion terms (e.g. $D_{\|}\Phi$ channels) that dephase off–diagonal sectors even faster. Write the block decomposition $\rho = \sum_{ij} \rho_{ij}$ with $\rho_{ij} \equiv \Pi_i \rho \Pi_j$. Then

$$\begin{aligned} \dot{\rho}_{ij} &= -i\Pi_i[H_{\mathrm{eff}}, \rho]\Pi_j - \tfrac{1}{2}\,\Gamma_{ij}\,\rho_{ij} + \mathcal{O}(\text{diffusive})\,, \qquad i \neq j, \\ \Gamma_{ij} &\equiv \int d^3x \,\Big(\gamma_i(x) + \gamma_j(x)\Big) \;\geq\; 0. \end{aligned} \tag{1.49}$$

Thus off–diagonal blocks decay with rate Γ_{ij}, while the populations $p_i(t) \equiv \mathrm{Tr}\,\rho_{ii}(t)$ obey a kinetic equation

$$\dot{p}_i(t) \;=\; \Phi_i(t) \;-\; \kappa_i\, p_i(t) \;+\; \sum_{j\neq i}\Big(W_{i\leftarrow j}\, p_j - W_{j\leftarrow i}\, p_i\Big), \tag{1.50}$$

where: (i) Φ_i is the *injection/flooding current* into sector i from the system–instrument coupling; (ii) κ_i is a leakage/relaxation rate inside the instrument sector; (iii) W are weak inter–sector transfer rates (instrument crosstalk; typically negligible after design). The *locking time* τ_{lock} is the inverse spectral gap of the GKSL generator restricted to block–diagonal states, giving exponential approach to a unique steady state whenever $\Gamma_{ij} > 0$ and $\kappa_i \geq 0$:

$$\big\|\rho(t) - \rho_\infty\big\|_1 \;\leq\; C\, e^{-t/\tau_{\mathrm{lock}}}, \qquad \tau_{\mathrm{lock}}^{-1} \;\gtrsim\; \min_{i\neq j}\Gamma_{ij} \;\wedge\; \min_i \kappa_i. \tag{1.51}$$

1.5.2 Stationary energy–share derivation of Born weights

We now compute the steady populations $P_i \equiv \lim_{t\to\infty} p_i(t)$ from an *energy continuity* perspective. Define the coarse–grained spectral energy in sector i,

$$E_i(t) \;\equiv\; \int d^3x\, \varepsilon_i(x,t), \qquad \varepsilon_i(x,t) \;\equiv\; \mathrm{Tr}\Big(\mathcal{H}_i(x)\,\rho_{ii}(t)\Big), \tag{1.52}$$

where $\mathcal{H}_i(x)$ is the instrument's pointer–channel energy density associated with Π_i. Under the dynamics (1.48), E_i obeys

$$\dot{E}_i(t) \;=\; \Phi_i^{(E)}(t) \;-\; \kappa_i^{(E)}\, E_i(t) \;+\; (\text{small } W \text{ terms}). \tag{1.53}$$

The *flooding power* $\Phi_i^{(E)}$ is obtained to leading order by a Golden–Rule evaluation over apparatus correlators (Sec. 1.4):

$$\Phi_i^{(E)} \;=\; 2\pi \int d\omega\; \mathcal{J}_i(\omega) \sum_\alpha \Big|\big\langle i,\omega;\alpha\big|\,\widehat{\mathcal{O}}_{\mathrm{sys}}\,\big|\psi\big\rangle\Big|^2, \tag{1.54}$$

where $\mathcal{J}_i(\omega)$ is the (positive) spectral density of the i-th pointer channel, $|\psi\rangle$ the initial system state, and α labels residual instrument states.

Channel–symmetry hypothesis (CS). Assume that over the operating band B:

(CS1) $\mathcal{J}_i(\omega) \equiv \mathcal{J}(\omega)$ is i–independent (*isoresponsive channels*);

(CS2) $\widehat{\mathcal{O}}_{\mathrm{sys}}$ is diagonal in $\{\Pi_i\}$ up to state–independent form factors over B.

Then (1.54) reduces to

$$\Phi_i^{(E)} \;=\; \mathcal{K}\,\big\|\Pi_i\,|\psi\rangle\big\|^2, \qquad \mathcal{K} \;\equiv\; 2\pi\int_B d\omega\, \mathcal{J}(\omega)\,|\mathsf{F}(\omega)|^2, \tag{1.55}$$

with F a common form factor. In the steady state of (1.53) (neglecting W),

$$E_i^\infty \;=\; \frac{\Phi_i^{(E)}}{\kappa_i^{(E)}}, \qquad P_i \;\equiv\; \lim_{t\to\infty} p_i(t) \;=\; \frac{E_i^\infty}{\sum_j E_j^\infty} \;=\; \frac{\Phi_i^{(E)}/\kappa_i^{(E)}}{\sum_j \Phi_j^{(E)}/\kappa_j^{(E)}}. \tag{1.56}$$

If, in addition, the instrument sectors have comparable impedance/leakage (*balanced* $\kappa_i^{(E)} \equiv \kappa^{(E)}$), then

$$P_i \;=\; \frac{\Phi_i^{(E)}}{\sum_j \Phi_j^{(E)}} \;=\; \frac{\big\|\Pi_i\,|\psi\rangle\big\|^2}{\sum_j \big\|\Pi_j\,|\psi\rangle\big\|^2} \;=\; \big\|\Pi_i\,|\psi\rangle\big\|^2. \tag{1.57}$$

Writing $|\psi\rangle = \sum_i |\psi_i\rangle$, $|\psi_i\rangle \equiv \Pi_i|\psi\rangle$, and introducing $\Psi_i(x)$ as the resolved amplitude in i, one recovers the *Born weights* as stationary energy shares,

$$P_i \;=\; \frac{\int d^3x\,\varepsilon_i(x)}{\sum_j \int d^3x\,\varepsilon_j(x)} \;\equiv\; \int d^3x\,|\Psi_i(x)|^2. \tag{1.58}$$

Theorem 1.1 (Born limit via stationary energy share)**.** *Under (CS1)–(CS2), with dephasing rates Γ_{ij} large enough to justify secular reduction and balanced leakage $\kappa_i^{(E)} = \kappa^{(E)}$, the GKSL dynamics (1.48) drives $p_i(t) \to P_i$ with P_i given by (1.57). Convergence is exponential with time constant $\tau_{\rm lock}$ in (1.51).*

Sketch. Fast dephasing (Γ_{ij}) block–diagonalizes ρ. The energy balance (1.53) closes on $\{E_i\}$ with sources (1.55). The unique fixed point yields (1.56); balance of leakages cancels $\kappa^{(E)}$, proving (1.57).

1.5.3 Deviations and bounds

Relaxing (CS) and allowing finite–rate effects, write

$$P_i \;=\; \frac{(\Phi_i^{(E)}/\kappa_i^{(E)})}{\sum_j(\Phi_j^{(E)}/\kappa_j^{(E)})} \;=\; \|\Pi_i\psi\|^2 \;+\; \delta P_i, \qquad \sum_i \delta P_i = 0. \tag{1.59}$$

Define channel–asymmetry and finite–rate parameters

$$\Xi_J \;\equiv\; \sup_{i,j} \frac{\|\mathcal{J}_i - \mathcal{J}_j\|_B}{\overline{\mathcal{J}}}, \quad \Xi_\kappa \;\equiv\; \max_i \left|\frac{\kappa_i^{(E)}}{\overline{\kappa}^{(E)}} - 1\right|, \quad \epsilon_{\rm sec} \;\equiv\; \max_{i\neq j} \frac{\|H_{\rm eff}\|}{\Gamma_{ij}}, \tag{1.60}$$

with $\overline{\mathcal{J}}$ and $\overline{\kappa}^{(E)}$ band averages, and $\epsilon_{\rm sec}$ quantifying residual off–diagonal coherences. Then for some $C = O(1)$,

$$\max_i |\delta P_i| \;\le\; C\Big(\Xi_J + \Xi_\kappa + \epsilon_{\rm sec}\Big). \tag{1.61}$$

Equivalently, in terms of physical rates,

$$|\delta P_i| \;=\; \mathcal{O}\!\left(\frac{\Delta\mathcal{J}}{\overline{\mathcal{J}}}\right) + \mathcal{O}\!\left(\frac{\Delta\kappa^{(E)}}{\overline{\kappa}^{(E)}}\right) + \mathcal{O}\!\left(\frac{\|H_{\rm eff}\|}{\min_{i\neq j}\Gamma_{ij}}\right), \qquad \Rightarrow \quad \delta P_i \xrightarrow[\text{good design}]{} 0. \tag{1.62}$$

In practice we will report experimental upper bounds on (Ξ_J, Ξ_κ) as *instrument–calibration priors.*

1.5.4 The Zeno and anti–Zeno regimes

Let the measurement/probe be implemented by a pulse train with envelope $g(t)$. Introduce a *colored Zeno parameter*

$$\mathcal{Z} \;\equiv\; \int_{-\infty}^{+\infty} d\omega\; \mathcal{J}(\omega)\, n_{\rm pulse}(\omega)\, S_{\rm sys}(\omega), \tag{1.63}$$

where: $\mathcal{J}(\omega)$ is the apparatus spectral density; $n_{\rm pulse}(\omega)$ the spectral occupancy of the pulse train (for Δt spacing, $n_{\rm pulse}(\omega) \propto \mathrm{sinc}^2(\omega\Delta t/2)$ times a comb); $S_{\rm sys}(\omega)$ the system transition spectrum (from $H_{\rm eff}$). For weak coupling, transition rates scale as $\Gamma(\Delta t) \propto \mathcal{Z}$, hence:

$$\text{Zeno suppression } (\Gamma\downarrow) \iff \text{spectral mismatch: } \mathrm{supp}\, n_{\rm pulse} \cap \mathrm{supp}\, S_{\rm sys} \text{ small}, \tag{1.64}$$

$$\text{anti–Zeno enhancement } (\Gamma\uparrow) \iff \text{spectral alignment: } \mathrm{supp}\, n_{\rm pulse} \text{ covers peaks of } S_{\rm sys} \text{ through } \mathcal{J}. \tag{1.65}$$

Quantitatively, for a single dominant Bohr frequency ω_0 and narrowband pulses,

$$\Gamma(\Delta t) \;\simeq\; 2\pi\, \mathcal{J}(\omega_0)\, S_{\rm sys}(\omega_0)\, n_{\rm pulse}(\omega_0), \qquad \frac{\Gamma(\Delta t)}{\Gamma_0} \;\approx\; \frac{n_{\rm pulse}(\omega_0)}{n_{\rm pulse}^{(\rm WB)}(\omega_0)}, \tag{1.66}$$

with Γ_0 the wide–band continuous–measurement rate. Thus *colored* pulse trains allow tunable Zeno/anti–Zeno control by shaping $n_{\rm pulse}(\omega)$.

Summary. In the spectral–medium picture, measurement *is* an energy–biased open evolution: viscous channels dephase and lock the state into pointer sectors, while stationary shares reproduce Born weights when channels are symmetric and balanced. Deviations are bounded by controlled asymmetries and finite–rate effects; Zeno physics is naturally encoded by the overlap of the probe's spectral profile with the system spectrum through the apparatus density $\mathcal{J}(\omega)$.

1.5.5 The Damascus Interpretation: Collapse as Emergent Spectral Flooding

Motivating idea. In the *Damascus* interpretation, the apparent "collapse of the wavefunction" is *not* a primitive postulate but a **dynamical** consequence of open spectral evolution: a measurement device—a dense, viscous sector of the medium—opens dissipative channels that *flood* the system's spectral modes and *lock* the state into a stable pointer eigenmode. Probabilities arise as stationary energy shares in the long-time limit.

Generator and causal locality. Starting from the spectral GKSL (Lindblad) generator (Sec. 1.4.3),

$$\dot\rho = -i[H_{\rm eff}, \rho] - \sum_a \gamma_a \int d^3x \left(L_a \rho L_a^\dagger - \tfrac{1}{2}\{L_a^\dagger L_a, \rho\} \right), \qquad L_a = L_a[\eta, \rho, \chi^\mu; D_\mu], \tag{1.67}$$

the kernels of L_a have support *inside* the effective causal cone of $g_{\rm eff}$, ensuring no superluminal influence. The rates $\gamma_a(\mathcal{S}) \geq 0$ are state-dependent and respect complete positivity (CP).

Decoherence vs. locking. Choose pointer projectors $\{\Pi_i\}$ that diagonalize the strongly damped instrument couplings (Sec. 1.5.1). In this basis, off-diagonal entries decay as

$$\frac{d}{dt}\rho_{ij} = -\Gamma_{ij}\,\rho_{ij} + \cdots, \qquad \Gamma_{ij} = \frac{1}{2}\sum_a \gamma_a \int d^3x \left(\ell_a^{(i)}(x) - \ell_a^{(j)}(x)\right)^2 \geq 0, \tag{1.68}$$

with $L_a\Pi_i = \ell_a^{(i)}\Pi_i$ (no sum). The *locking* time is set by the spectral gap of the Liouvillian $\mathcal{L}$ restricted to block-diagonal states,

$$\tau_{\rm lock} \simeq \lambda_\star^{-1}, \qquad \lambda_\star = \min\{\mathrm{Re}\ \lambda > 0 :\ \lambda \in \mathrm{Spec}(\mathcal{L})\}, \tag{1.69}$$

and obeys $\tau_{\rm lock} \geq \max_{i\neq j}\Gamma_{ij}^{-1}$. Operationally, collapse is *finite-rate* and *causal*.

Born weights as stationary energy shares. Let the instrument sector decompose $\mathcal{H} = \bigoplus_i \mathcal{H}_i$ with Π_i the corresponding projectors. In the long-time regime $t \gg \tau_{\rm lock}$ one finds

$$P_i(t) = \mathrm{Tr}(\Pi_i\rho_t) \ \longrightarrow\ \frac{\int d^3x\,\varepsilon_i(x)}{\sum_j \int d^3x\,\varepsilon_j(x)} \equiv \int |\Psi_i(x)|^2\, d^3x, \tag{1.70}$$

where $\varepsilon_i(x)$ is the spectral energy density associated with the locked pointer eigenmode i. Thus the Born rule is the *stationary, long-time limit* of spectral flooding.

Arrow of time and entropy production. With $\eta > 0$ one has nonnegative entropy production (Sec. 1.4.4),

$$\dot{S}_{\rm vN} \ = \ \sum_a \gamma_a\,\mathrm{Tr}\Big(L_a^\dagger L_a\,\rho - L_a\rho L_a^\dagger\Big) \ \geq 0, \tag{1.71}$$

selecting a dynamical arrow for measurement processes without extra postulates.

Controlled deviations. Finite-rate/colored driving produces small, testable corrections,

$$\delta P_i = \mathcal{O}\Big(\frac{\tau_{\rm dyn}}{\tau_{\rm dec}},\ \frac{\tau_{\rm dyn}}{\tau_{\rm lock}},\ \mathcal{Z}^{-1}\Big), \qquad \mathcal{Z}(\omega) = \int d\omega\,\mathcal{J}(\omega)\,n_{\rm pulse}(\omega), \tag{1.72}$$

with $\tau_{\rm dyn}$ a system timescale and $\mathcal{Z}$ the colored-Zeno parameter (Sec. 1.5.4).

Empirical targets. (i) *Phase-only* double-slit: visibility loss with measurable phase shifts at fixed counts (Sec. 1.7.1); (ii) *Colored Zeno* crossovers in guided matter/photonic waves (Sec. 1.7.2); (iii) *Thread cutting* in entanglement by local impairment of χ^μ (Sec. 1.6.3).

1.5.6 Damascus vs. Copenhagen: a focused comparison

- **Ontology.** *Damascus*: a physical spectral medium; "wavefunction" encodes a real, distributed state (§1.2). *Copenhagen*: the wavefunction is chiefly an information carrier/predictive tool.
- **Collapse.** *Damascus*: derived, finite-rate, causal (GKSL with medium–instrument couplings); *Copenhagen*: instantaneous projection postulate.

- **Born rule.** *Damascus*: stationary energy-share limit (1.70); small, quantifiable deviations possible in non-asymptotic regimes. *Copenhagen*: fundamental axiom with no underlying dynamical derivation.
- **Causality/no signalling.** *Damascus*: ensured by kernel support inside the g_{eff} cone (no superluminal influence). *Copenhagen*: no-signalling guaranteed by the formal projection plus tensor-product structure.
- **Pointer basis.** *Damascus*: selected by the dissipative instrument sector (eigenbasis of L_a); *Copenhagen*: chosen by the "classical" apparatus without a microscopic selection rule.
- **Testability.** *Damascus*: predicts rate- and spectrum-dependent signatures (phase-only fringes, colored Zeno, thread-cut entanglement). *Copenhagen*: identical outcome statistics, but no quantitative dependence on apparatus spectra beyond decoherence heuristics.
- **Unification.** *Damascus*: same medium governs quantum foundations, cosmology, and emergent geometry; *Copenhagen*: interpretation-layer separate from dynamical origin.

Aspect	**Damascus (spectral)**	**Copenhagen**
Wavefunction	Real distributed state of medium	Epistemic/predictive tool
Collapse	Emergent, finite-rate (GKSL), causal	Instantaneous postulate
Born rule	Stationary energy share (derivation)	Axiom
Pointer basis	Eigenbasis of L_a (device-driven)	Chosen by classical apparatus
Causality	Kernel support $\subset$ cone of g_{eff}	Ensured formally by projection
Deviations	Small, rate/spectrum-controlled δP_i	None (exact axiom)
Empirical knobs	$\gamma_a(\mathcal{S})$, η, spectral coloring	Not specified
Scope	Unified with cosmology/geometry	Interpretation adjunct

Table 1.1: Focused contrasts between the Damascus and Copenhagen interpretations.

1.6 Entanglement as Structural Nonseparability

We define *entanglement* as the failure of factorization of states and correlators across spatial/composite partitions *due to* the connectivity induced by the internal director field χ^μ and the medium's spectral couplings. Operationally, entanglement is diagnosed by (i) reduced-state nonfactorization on bipartitions aligned with the χ^μ-induced graph, (ii) Bell/CHSH violations up to the Tsirelson bound in spacelike configurations, and (iii) controlled degradation (without signalling) under local weakening or cutting of χ-threads.

1.6.1 χ^μ-threads and bipartitions

Let Σ be a Cauchy slice for g_{eff}, and discretize it (conceptually) into cells $\{X_i\}$ of linear size ℓ with $\xi \ll \ell \ll$ (instrument scale). Define a weighted, directed graph

$$\mathcal{G}_\chi = (\mathcal{V}, \mathcal{E}, w), \qquad \mathcal{V} = \{X_i\}, \quad (i \to j) \in \mathcal{E} \text{ iff } w_{ij} > 0, \tag{1.73}$$

with edge-weights

$$w_{ij} \;\equiv\; \int_{X_i\times X_j} d^3x\, d^3y\, \Big[\zeta_\parallel(\mathcal{S})\,\hat\chi^{(0)}\cdot\nabla_x\Big]\, G_\chi(x,y)\,\Big[\hat\chi^{(0)}\cdot\nabla_y\zeta_\parallel(\mathcal{S})\Big], \tag{1.74}$$

where G_χ is the Green kernel of the χ-sector linearized operator,

$$\mathsf{L}_\chi\, G_\chi(\cdot,y) = \delta(\cdot - y), \qquad \mathsf{L}_\chi \equiv -\nabla_i\Big(Z_\perp^{ij}\nabla_j\Big) - \partial_\parallel\Big(Z_\parallel\,\partial_\parallel\Big) + m_\chi^2(\mathcal{S}), \tag{1.75}$$

and $\partial_\parallel \equiv \hat\chi^{(0)}\cdot\nabla$. A bipartition (A,B) is a cut of $\mathcal{G}_\chi$, i.e. $\mathcal{V} = A \sqcup B$. Given the full density operator ρ on the system–instrument Hilbert space $\mathcal{H} = \mathcal{H}_A \otimes \mathcal{H}_B$ (constructed in Sec. 1.3), the reduced states are $\rho_A = \mathrm{Tr}_B\,\rho$ and $\rho_B = \mathrm{Tr}_A\,\rho$.

Structural nonseparability. We call a state structurally nonseparable across (A,B) if either (i) $\rho \neq \sum_k p_k \rho_A^{(k)} \otimes \rho_B^{(k)}$, or (ii) there exist local bounded operators O_A, O_B such that

$$\Big|\mathrm{Tr}\,\rho\, O_A\otimes O_B - \mathrm{Tr}(\rho_A O_A)\,\mathrm{Tr}(\rho_B O_B)\Big| \;\geq\; \mathcal{F}(A,B;\mathcal{G}_\chi) \;>\; 0, \tag{1.76}$$

where $\mathcal{F}$ is a monotone increasing functional of the (weighted) number of disjoint χ-threads across the cut (e.g. the min-cut capacity $\mathrm{cap}_{\min}(A{:}B)$ computed from w_{ij}). In the Gaussian/quadratic sector this reduces to *two-point* function bounds; for generic states, (1.76) is understood via entanglement witnesses.

Continuum proxy. In the continuum, define a *thread propagator norm*

$$\|G_\chi\|_{A\to B} \;\equiv\; \sup_{\substack{f\in L^2(A),\, g\in L^2(B)\\ \|f\|=\|g\|=1}} \Big|\int_{A\times B} d^3x\, d^3y\; f(x)\, G_\chi(x,y)\, g(y)\Big|. \tag{1.77}$$

Then any Gaussian state with covariance C satisfies an upper bound on the logarithmic negativity $\mathcal{E}_N(A:B)$,

$$\mathcal{E}_N(A:B) \;\leq\; C(\mathsf{K},\mathsf{Z}^{ij})\,\|G_\chi\|_{A\to B}, \tag{1.78}$$

with C set by quadratic coefficients (Sec. 1.3). This makes explicit the *directional* (anisotropic) character of structural entanglement inherited from $\hat\chi^{(0)}$.

1.6.2 Bell/CHSH and no–signalling

Consider two spacelike separated instrument regions A, B (with respect to g_{eff}). Local instruments are generated by GKSL operators with compact causal support (Sec. 1.4):

$$\mathcal{L}_A(\cdot) = \sum_\alpha \int_A d^3x\, \gamma_\alpha(x)\Big(L_\alpha(x)\,(\cdot)\,L_\alpha^\dagger(x) - \tfrac{1}{2}\{L_\alpha^\dagger(x)L_\alpha(x),(\cdot)\}\Big), \tag{1.79}$$

and analogously $\mathcal{L}_B$ on B. Microcausality in the effective theory implies

$$\Big[L_\alpha(x),\, L_\beta(y)\Big] = 0 = \Big[L_\alpha(x),\, L_\beta^\dagger(y)\Big], \qquad \forall\, x\in A,\; y\in B \text{ spacelike}. \tag{1.80}$$

Therefore the total generator splits and *commutes*,

$$\mathcal{L}_{A\cup B} = \mathcal{L}_A + \mathcal{L}_B, \qquad [\mathcal{L}_A,\mathcal{L}_B] = 0, \tag{1.81}$$

which ensures *no–signalling* in the reduced dynamics:

$$\frac{d}{dt}\rho_A(t) = \mathrm{Tr}_B\Big[(\mathcal{L}_A+\mathcal{L}_B)\rho(t)\Big] = \mathcal{L}_A\Big(\rho_A(t)\Big), \tag{1.82}$$

independent of any choice of settings on B (and vice versa).

CHSH/Tsirelson. Let A_0, A_1 and B_0, B_1 be dichotomic, norm–bounded local observables ($\|A_s\| \le 1$, $\|B_t\| \le 1$), supported in A and B respectively, and commuting across spacelike regions by (1.80). The CHSH operator is

$$\widehat{\mathcal{B}} \equiv A_0 \otimes (B_0 + B_1) + A_1 \otimes (B_0 - B_1). \tag{1.83}$$

The standard operator inequality (Tsirelson) gives

$$\|\widehat{\mathcal{B}}\| \;\le\; 2\sqrt{2}, \qquad \Rightarrow \quad \mathcal{S} \;\equiv\; \left|\operatorname{Tr} \rho\, \widehat{\mathcal{B}}\right| \;\le\; 2\sqrt{2}, \tag{1.84}$$

with classical (local–hidden–variable) bound 2. Within our framework, *maximal* quantum violation $\mathcal{S} = 2\sqrt{2}$ is achieved when: (i) the state is a *structural Bell pair* created by a χ-mediated pairing Hamiltonian on a two–mode subspace,

$$H_{\text{pair}} = \lambda\,(a_A^\dagger a_B^\dagger + a_A a_B), \qquad \lambda > 0, \quad [a_A, a_B^\dagger] = 0, \tag{1.85}$$

(ii) the modes are supported on disjoint spacelike regions, and (iii) the measurement settings are orthogonal in the two–mode Bloch sphere. Anisotropy enters through reduced visibilities $\mathcal{V}(\theta)$ if the local readouts are aligned/misaligned with $\hat{\chi}^{(0)}$, yielding

$$\mathcal{S}(\theta) = 2\sqrt{2}\,\mathcal{V}(\theta), \qquad 0 \le \mathcal{V}(\theta) \le 1, \tag{1.86}$$

where $\mathcal{V}(\theta)$ decreases with the angle between the instrument axes and the dominant thread direction, and with decoherence from local η.

1.6.3 Cutting threads: degradations without signalling

Local manipulation of the director sector—e.g. increasing the viscous damping η in a compact region $R \subset A$, or applying a "scrambling" field that reduces $Z_\parallel$—modifies L_χ in (1.75) to

$$\mathsf{L}_\chi^{(R)} = \mathsf{L}_\chi + \Delta\mathsf{L}_R, \qquad \Delta\mathsf{L}_R \ge 0 \;\text{ supported in } R, \tag{1.87}$$

and consequently the propagator to $G_\chi^{(R)}$. By the resolvent identity,

$$G_\chi^{(R)} \;=\; G_\chi - G_\chi\,\Delta\mathsf{L}_R\,G_\chi^{(R)} \quad \Rightarrow \quad \|G_\chi^{(R)}\|_{A\to B} \;\le\; \frac{\|G_\chi\|_{A\to B}}{1 + \lambda_{\min}(\Delta\mathsf{L}_R)\,\|G_\chi\|_{A\to A}}, \tag{1.88}$$

so that thread capacity across the cut decreases monotonically with the "impedance" $\lambda_{\min}(\Delta\mathsf{L}_R)$.

Degradation of nonlocal correlations. For any bounded local observables O_A, O_B,

$$\Big|\,\langle O_A \otimes O_B\rangle_{G_\chi^{(R)}} - \langle O_A \otimes O_B\rangle_{G_\chi}\,\Big| \;\le\; C_O\,\Big(\|G_\chi\|_{A\to B} - \|G_\chi^{(R)}\|_{A\to B}\Big), \tag{1.89}$$

and, in particular, the CHSH parameter obeys

$$\mathcal{S}^{(R)} \;\le\; 2\sqrt{2}\,\mathcal{V}^{(R)}(\theta), \qquad \mathcal{V}^{(R)}(\theta) \;\le\; \mathcal{V}(\theta), \tag{1.90}$$

with $\mathcal{V}^{(R)}(\theta)$ determined by $G_\chi^{(R)}$. Crucially, *no signalling* is preserved: the local reduced dynamics in B is unchanged by $\Delta\mathsf{L}_R$ in A,

$$\frac{d}{dt}\rho_B(t) = \mathcal{L}_B\Big(\rho_B(t)\Big), \qquad \forall\,\Delta\mathsf{L}_R \text{ supported in spacelike } A, \tag{1.91}$$

by the same commutation/splitting argument (1.81)–(1.82).

Exponential attenuation model. For approximately homogeneous media along a geodesic γ crossing the cut, the thread norm decays as

$$\|G_\chi^{(R)}\|_{A\to B} \;\simeq\; \|G_\chi\|_{A\to B}\,\exp\!\Big[-\int_\gamma ds\,\kappa_\chi(s)\Big], \qquad \kappa_\chi(s) \;\propto\; \sqrt{\eta(s)\,Z_\parallel^{-1}(s)}\,, \tag{1.92}$$

giving a simple, testable law for the suppression of Bell correlators under local χ-impairment. Equation (1.92) provides an experimental *dial* to probe the structural origin of entanglement without enabling superluminal control.

Summary. Entanglement is rendered concrete as connectivity by χ-threads in a causal medium. Bell violations up to the Tsirelson bound coexist with strict no–signalling because local GKSL generators commute across spacelike regions. Local "cuts" reduce the thread capacity—and thus the achievable nonlocal correlations—monotonically, offering a direct, falsifiable handle on the structural (rather than axiomatic) origin of quantum nonseparability.

1.7 Case Studies: Double Slit and Colored Zeno

We work out two concrete, falsifiable scenarios where the spectral–medium interpretation departs in emphasis (not in predictions) from the standard axiomatization: (i) a *phase–only tagged* double–slit where the interference *shifts* at fixed total counts while the visibility is set by spectral dephasing Γ_ϕ; and (ii) a *colored Zeno* protocol where the survival/transition rate is governed by the spectral overlap between an engineered pulse filter and the environmental (aether–instrument) spectral density $\mathcal{J}(\omega)$.

1.7.1 Double slit with phase-only tagging

Consider two narrow slits S_A, S_B separated by d and a screen at distance L in the aether rest frame u^μ. Let $\mathbf{k} = k\,\hat{\mathbf{z}}$ be the central wavevector of the incoming packet and let θ denote the angle between the local propagation direction and the internal director $\hat{\chi}^{(0)}$ (anisotropy axis). In the Fraunhofer regime, the complex field at a screen coordinate X (transverse to $\hat{\mathbf{z}}$) reads

$$\Psi(X) \;=\; \Psi_A(X) \;+\; e^{i\delta_{\rm tag}}\,e^{-\frac{1}{2}\Gamma_\phi(\theta,k)\,\tau(X)}\,\Psi_B(X), \tag{1.93}$$

where $\delta_{\rm tag}$ is a *pure phase* imprinted locally near S_B (e.g. by a thin, lossless phase plate that alters the spectral phase of Φ without absorption), $\tau(X) \simeq L/v_g(\theta)$ is the flight time with group speed $v_g(\theta)$ of the relevant mode, and Γ_ϕ is the *spectral dephasing* rate generated by the viscosity sector (Sec. 1.4), parametrized at leading order as

$$\Gamma_\phi(\theta,k) \;\simeq\; \frac{\eta}{\kappa}\left(c_\parallel^{-2}\cos^2\theta + c_\perp^{-2}\sin^2\theta\right)k^2 \;\equiv\; \frac{\eta}{\kappa}\,\alpha(\theta)\,k^2, \tag{1.94}$$

with (κ,η) the spectral stiffness/viscosity, and $c_\parallel, c_\perp$ the directional mode speeds (Vol. II, Sec. **??**). The intensity on the screen is then

$$I(X) = |\Psi_A|^2 + |\Psi_B|^2 + 2\,|\Psi_A||\Psi_B|\,\mathcal{V}(\theta,k)\,\cos\Big(\Delta\varphi(X) + \delta_{\rm tag}\Big), \tag{1.95}$$

$$\mathcal{V}(\theta,k) \equiv \exp\Big[-\tfrac{1}{2}\Gamma_\phi(\theta,k)\,\tau(X)\Big] \times \mathcal{V}_0, \qquad \Delta\varphi(X) \simeq \frac{2\pi}{\lambda_{\rm eff}}\frac{d\,X}{L} + \varphi_0, \tag{1.96}$$

where $\mathcal{V}_0 \le 1$ is the baseline visibility from source coherence and finite slit width, and $\lambda_{\rm eff} = 2\pi/k$.

Phase shifts at fixed total counts. Because the tag is *lossless*, the total flux is conserved:

$$\int dX\, I(X) = \int dX\, (|\Psi_A|^2 + |\Psi_B|^2) \quad \text{(the fringe term averages to zero over many periods).} \tag{1.97}$$

Hence *phase-only tagging* produces a rigid shift of the fringes,

$$X_{\max} \;\mapsto\; X_{\max} + \Delta X, \qquad \Delta X \;=\; \frac{\lambda_{\text{eff}} L}{2\pi d}\, \delta_{\text{tag}}, \tag{1.98}$$

at essentially unchanged integrated counts, while the *visibility* is governed by the spectral factor $\mathcal{V}(\theta, k)$.

Anisotropy and tests. Varying the apparatus orientation relative to $\hat{\chi}^{(0)}$ yields a predictable modulation of $\mathcal{V}$ through $\alpha(\theta)$ in (1.94). At fixed (k, L, d) one expects

$$\frac{\ln \mathcal{V}(\theta_1)}{\ln \mathcal{V}(\theta_2)} \;\simeq\; \frac{\alpha(\theta_1)}{\alpha(\theta_2)} \;=\; \frac{c_{\parallel}^{-2}\cos^2\theta_1 + c_{\perp}^{-2}\sin^2\theta_1}{c_{\parallel}^{-2}\cos^2\theta_2 + c_{\perp}^{-2}\sin^2\theta_2}, \tag{1.99}$$

a clean diagnostic of directional spectral transport. Moreover, the joint prediction *(i)* conserved flux with *(ii)* rigid phase shift (1.98) and *(iii)* exponential visibility law (1.96) distinguishes *phase–only* from absorptive tagging.

1.7.2 Colored Zeno in waveguides / cold atoms

We formalize the *colored Zeno* idea introduced in Sec. 1.5.4: the effective decay/survival rate of a driven–monitored system is the *overlap* of an environmental spectral density $\mathcal{J}(\omega)$ (set by the aether–instrument composite) with a *measurement filter* $F(\omega)$ determined by the pulse sequence.

Filter–overlap law. For a two–level transition at frequency ω_0 (or a narrow band), and a control sequence with pulse times $\{t_j\}_{j=0}^{N}$ ($t_0 = 0$, $t_N = T$), the leading weak–coupling transition rate reads

$$\Gamma_{\text{eff}}(T, \{t_j\}) = \int_0^\infty d\omega\; \mathcal{J}(\omega)\, F(\omega; T, \{t_j\}), \qquad F(\omega) = \frac{1}{2\pi}\left| \sum_{j=0}^{N-1} (-1)^j \left(e^{i\omega t_{j+1}} - e^{i\omega t_j}\right)\right|^2, \tag{1.100}$$

where F is the standard filter function of dynamical decoupling (with the $(-1)^j$ toggling due to π pulses). In our medium, $\mathcal{J}(\omega)$ is *colored* by spectral transport and viscosity:

$$\mathcal{J}(\omega) \;\propto\; \frac{\omega^s}{\left[1 + (\omega/\omega_c)^\nu\right]}\, \mathcal{A}(\theta), \qquad \mathcal{A}(\theta) = \beta_{\parallel}\cos^2\theta + \beta_{\perp}\sin^2\theta, \tag{1.101}$$

with $s \in \{1, 3\}$ (Ohmic–to–super–Ohmic), cutoff ω_c set by the instrument/medium correlation time, and anisotropy contained in $\mathcal{A}(\theta)$ via χ^μ.

Define the *colored Zeno functional*

$$\mathcal{Z}_N(T) \;\equiv\; \frac{\Gamma_{\text{eff}}(T, \{t_j\})}{\Gamma_{\text{nat}}}, \qquad \Gamma_{\text{nat}} = \int_0^\infty d\omega\, \mathcal{J}(\omega)\, F_0(\omega; T), \tag{1.102}$$

where F_0 is the *free* (no pulses) filter ($N = 0$). Then:

$$\text{Zeno regime:} \quad \mathcal{Z}_N(T) < 1 \quad \Leftrightarrow \quad F(\omega) \text{ suppresses overlap near peaks of } \mathcal{J}, \tag{1.103}$$

$$\text{anti–Zeno:} \quad \mathcal{Z}_N(T) > 1 \quad \Leftrightarrow \quad F(\omega) \text{ enhances overlap (aligns side–lobes with } \mathcal{J}). \tag{1.104}$$

Waveguide design (photons/magnons). In a 1D/2D waveguide with a band edge at ω_b, choose a CPMG sequence ($t_{j+1} - t_j = \tau$) so that the main notch of $F(\omega)$ sits on resonance:

$$\tau_{\mathrm{Z}} \simeq \frac{\pi}{\omega_0}, \qquad F(\omega_0) \approx 0 \;\Rightarrow\; \Gamma_{\text{eff}} \ll \Gamma_{\text{nat}} \quad \text{(Zeno suppression)}. \tag{1.105}$$

For anti–Zeno, deliberately align a side–lobe at a guided–mode peak ω_p by choosing τ_{aZ} such that the first maximum of F satisfies $F(\omega_p)$ large; then $\Gamma_{\text{eff}} > \Gamma_{\text{nat}}$. Anisotropy manifests via (1.101): rotating the device relative to $\hat{\chi}^{(0)}$ modulates $\mathcal{Z}_N$ by $\mathcal{A}(\theta)$, a clean handle on structural directionality.

Cold atoms in a lattice. Let ω_0 be a Raman splitting. Collisional/phononic noise gives an approximately Ohmic $\mathcal{J}(\omega)$ with cutoff $\omega_c \sim k_B T/\hbar$. Use Uhrig DD with $t_j = T \sin^2[\pi j/(2N+2)]$, whose filter has N zeros at low ω. The Zeno threshold is set by the bath correlation time $\tau_B \sim 1/\omega_c$:

$$T \ll \tau_B \;\Rightarrow\; \Gamma_{\text{eff}} \propto T^{N+1} \text{ (strong Zeno)}; \qquad T \gtrsim \tau_B \;\Rightarrow\; \text{crossover to anti–Zeno if } \partial_\omega \mathcal{J} > 0. \tag{1.106}$$

Closed–form small–τ asymptotics. For stroboscopic projections with spacing τ and smooth $\mathcal{J}$,

$$\Gamma_{\text{eff}}(\tau) \;=\; \int d\omega\, \mathcal{J}(\omega)\, \frac{\sin^2(\omega\tau/2)}{\pi(\omega/2)^2} \;=\; \frac{\tau}{2\pi}\, \mathcal{J}(0) \;+\; \mathcal{O}(\tau^3). \tag{1.107}$$

Thus $\Gamma_{\text{eff}} \to 0$ as $\tau \to 0$ when $\mathcal{J}(0)$ is finite (*universal Zeno*). For colored spectra that rise sharply beyond ω_c, increasing τ past $\tau_B \sim 1/\omega_c$ increases the weight in the high–frequency side–lobes and can induce the anti–Zeno crossover, in agreement with (1.103)–(1.104).

Summary and falsifiable knobs. The spectral formulation predicts: (i) *phase–only* tagging shifts fringes rigidly at fixed flux, while $\mathcal{V}$ obeys (1.94)–(1.96) with a clear θ–dependence; (ii) *colored Zeno* behavior is governed by the overlap integral (1.100), tunable by pulse timing and device orientation relative to $\hat{\chi}^{(0)}$. Both provide direct probes of the medium's spectral transport (κ, η) and anisotropy $(c_\parallel, c_\perp)$ without appealing to extraneous collapse postulates.

1.8 Consistency Gates and Restoration

We collect the foundational constraints that any spectral–quantum model must satisfy and state a *restoration* result: under coarse–graining and vanishing viscosity the theory reduces to an isotropic, single–clock EFT with standard adiabaticity and unitary microdynamics.

1.8.1 Mapping to S/G/TQ/D gates

(S) Structural stability and causality. Let $\mathcal{L}^{(2)}$ be the quadratic action of Sec. 1.3 with principal blocks $(\mathsf{K}, \mathsf{Z}^{ij})$ in the aether frame u^μ.

(S1) No ghosts: $\mathsf{K} \succ 0.$ (1.108)

(S2) Hyperbolicity/sub–luminality: $\lambda_{\max}\Big(\mathsf{K}^{-1/2}\,\mathsf{Z}^{ij}\hat{k}_i\hat{k}_j\,\mathsf{K}^{-1/2}\Big) = c^2(\theta) \;\in\; (0,1] \;\; \forall\,\hat{\mathbf{k}},$ (1.109)

(S3) Front causality: $v_{\rm front}(\hat{\mathbf{k}}) \le c_{\rm eff} = 1$ (the light cone of $g_{\rm eff}$). (1.110)

For the tensor sector h_{ij}:

$$0 < c_T^2 \le 1, \qquad |\delta c_T| \ll 1 \;\text{ in late–time media.} \tag{1.111}$$

(G) Gauge/emergent symmetry compatibility. Let $\{G^A\}$ be generators of emergent gauge symmetries (Vol. I, Ch. 4). The effective Hamiltonian and dissipators must be covariant:

$$[H_{\rm eff}, G^A] = 0, \qquad [L_a, G^A] = 0 \;\; \forall a, A, \tag{1.112}$$

and the Kossakowski kernel $\mathsf{C}_{ab}(x,y)$ defining the GKSL generator is positive semidefinite and gauge–invariant:

$$\int d^3x\, d^3y\, f_a^*(x)\, \mathsf{C}_{ab}(x,y)\, f_b(y) \;\ge 0, \quad \mathsf{C}_{ab} \;\text{ built from } (g_{\rm eff}, D_\mu, \mathcal{S}) \text{ and gauge scalars.} \tag{1.113}$$

(TQ) Topology/quantization integrity. For any topological charge $Q_{\rm top} = \int d^3x\, j^0_{\rm top}$ (winding/Hopf), admissible couplings preserve homotopy classes:

$$\frac{d}{dt}\,\mathrm{Tr}(\rho\, Q_{\rm top}) = -i\,\mathrm{Tr}\Big(\rho\,[Q_{\rm top}, H_{\rm eff}]\Big) \;-\; \sum_a \gamma_a\,\mathrm{Tr}\Big(\rho\,[Q_{\rm top}, L_a^\dagger L_a]\Big) = 0, \tag{1.114}$$

which holds if $[Q_{\rm top}, H_{\rm eff}] = [Q_{\rm top}, L_a] = 0$. Thus Lindblad channels may smear cores but cannot *unwind* defects.

(D) Data/diagnostics and positivity. (i) *Complete positivity* (CP) requires a positive Kossakowski matrix, ensuring physical reduced states. (ii) *No–signalling* follows from local kernels supported inside the $g_{\rm eff}$–cone:

$$\mathcal{D}[\rho] = \int_{\mathcal{J}^+(x)} d^3y\; \mathsf{C}_{ab}(x,y)\,\Big(L_a(y)\rho L_b^\dagger(y) - \tfrac{1}{2}\{L_b^\dagger(y)L_a(y), \rho\}\Big), \tag{1.115}$$

so spacelike–separated instrument algebras commute. (iii) *Entropy production:* for unital or thermal (KMS) dissipators tied to $\eta > 0$,

$$\dot{S}_{\rm vN} \;=\; -\mathrm{Tr}(\dot{\rho}\log\rho) \;\ge 0, \tag{1.116}$$

with equality only at fixed points of the semigroup. (iv) *Admissible anisotropy:* angle dependence $c_s(\theta) = c_0[1 + \epsilon\, P_2(\cos\theta)]$ with $|\epsilon| \ll 1$ in regimes constrained by isotropy tests (cf. Vol. II).

1.8.2 Restoration lemma across scales

Lemma (Single–clock, isotropic restoration). *Consider the spectral medium in a regime of (i) RG coarse–graining to scales $\ell \gg \xi$ (correlation length), (ii) vanishing effective viscosity in the system sector, $\eta_{\rm sys}\downarrow 0$ (no active instrument), and (iii) randomized director, $\langle\hat{\chi}\rangle = 0$ with finite variance. Then the reduced dynamics of fluctuations admits:*

1. *An isotropic quadratic action*

$$\mathcal{L}^{(2)} \;\to\; \tfrac{1}{2}\,\delta\mathbf{S}^{\top}\Big[\mathsf{K}\,\partial_t^2 + \mathsf{Z}\,\Delta + \mathsf{M}^2\Big]\delta\mathbf{S}, \qquad c_s^2 = \lambda_{\max}(\mathsf{K}^{-1/2}\mathsf{Z}\mathsf{K}^{-1/2}), \tag{1.117}$$

   with $c_s(\theta) \to c_s$ and $c_T \to 1$ (up to higher–order IR–irrelevant operators).

2. *Unitary microdynamics:* the GKSL generator collapses to Hamiltonian flow, $\dot{\rho} = -i[H_{\rm eff}, \rho]$, since $\gamma_a \propto \eta_{\rm sys} \to 0$ and the unital dissipator vanishes.

3. *Single–clock adiabaticity:* scalar perturbations obey a standard EFT with a unique freeze–out condition $k = aH/c_s$ and adiabatic mode conservation on super–Hubble scales; non–adiabatic sources are RG–irrelevant in this limit.

Sketch of proof. (i) Coarse–graining averages directional tensors: $\langle \mathsf{Z}^{ij}\rangle \propto \delta^{ij}$ and suppresses the anisotropic eigenvalues by $\sim (\xi/\ell)^{\Delta}$ with $\Delta > 0$ (irrelevant operator). (ii) Dissipative rates scale as $\gamma_a \sim \eta_{\rm sys}\,\mathcal{J}_a$; with no instrument channel, $\eta_{\rm sys} \to 0$ and CP–semigroup tends to the unitary group. (iii) In the isotropic, weakly varying background the curvature mode action reduces to the single–field form with conserved ζ, while tensor propagation approaches $c_T \to 1$ as curvature–sector higher–derivative terms renormalize to GR–like values.

Corollaries.

- *Born limit:* in the absence of active flooding channels, measurement reduces to projective updates consistent with standard quantum predictions; the "energy–share" derivation of Sec. 1.5.2 reproduces $P_i = \|\Psi_i\|^2$.
- *Tsirelson bound:* locality of the GKSL kernel and unitary reduction guarantee $\beta_{\rm CHSH} \le 2\sqrt{2}$; supra–quantum (PR–box) values are excluded by CP and cone–locality.
- *Restoration of symmetries:* gauge covariance (1.112) plus RG–irrelevance of anisotropy restore rotational/Lorentz–like symmetries of the $g_{\rm eff}$–cone in the IR.

Summary. The S/G/TQ/D gates enforce (i) healthy propagation inside the $g_{\rm eff}$ cone, (ii) compatibility with emergent gauge structure, (iii) preservation of topological sectors, and (iv) CP, locality, and entropy production under instrumental coupling. Under coarse–graining with $\eta\downarrow 0$, the theory *restores* to an isotropic, single–clock EFT with unitary microdynamics and standard adiabaticity—providing the precise bridge to conventional quantum theory in its empirical domain.

1.9 Interfaces and Outlook

This section ties the spectral–quantum framework to (i) emergent gauge symmetries from fiber isometries, (ii) concrete numerical/experimental protocols suitable for near–term benchmarks, and (iii) crisp, falsifiable signatures that distinguish our proposal from orthodox narratives.

1.9.1 Bridge to emergent gauge symmetries

Let $\mathcal{F}$ denote the internal spectral fiber whose coordinates $\{\varphi^A\}$ parametrize phases/polarizations assembled from $(\Phi, \chi^\mu, \dots)$. Assume a (possibly anisotropic) fiber metric $\gamma_{AB}(\mathcal{S})$ and an isometry group $\mathcal{G} = \mathrm{Iso}(\mathcal{F}, \gamma)$. Then:

Covariantization from fiber isometries. A local fiber rotation $U(x) \in \mathcal{G}$ acting on a field $\Psi(x) \in \mathcal{R}$ (a representation of $\mathcal{G}$) requires *spectral* covariant derivatives

$$D_\mu \Psi = \partial_\mu \Psi + \omega_\mu \, \Psi + A^I_\mu \, (T_I \cdot \Psi), \qquad A_\mu \equiv A^I_\mu T_I \in \mathfrak{g} = \mathrm{Lie}(\mathcal{G}), \tag{1.118}$$

where ω_μ is the emergent connection of Vol. I and A_μ is induced by spatial variation of the fiber frame. In coordinates,

$$A^I_\mu \;=\; \mathcal{U}^I{}_A(\varphi)\, \partial_\mu \varphi^A \;+\; \mathcal{O}(\partial\chi, \partial\Phi), \tag{1.119}$$

with $\mathcal{U}$ fixed by the Killing vectors on $(\mathcal{F}, \gamma)$. Thus gauge bosons arise as collective connections of the fiber isometries.

Ward identities and anomaly constraints. Let J^μ_I be the Noether currents for $\mathcal{G}$. Coupling to A^I_μ yields

$$\nabla_\mu J^\mu_I = \mathcal{A}_I[\mathcal{S}] \quad \text{(anomaly functional)}. \tag{1.120}$$

Consistency demands $\mathcal{A}_I \equiv 0$ (exact cancellation) or an emergent WZW/CS counterterm from the structural sector (Vol. I, App. C), both implementable through χ^μ–dependent topological densities. Hence the *gauge sector* is not postulated but *selected* by fiber geometry and topological consistency.

Role of the director χ^μ. The internal director controls (i) anisotropy of principal symbols, (ii) preferred frames for u^μ vs. fiber transport, and (iii) mixing angles in A_μ through (1.119). In isotropic restoration (Sec. 1.8.2), $\langle \hat{\chi} \rangle = 0$ and the gauge sector reduces to an isotropized connection compatible with single–clock adiabaticity.

1.9.2 Numerical protocols and datasets

We outline minimal simulators and benchmark datasets to test §1.4–1.7.

Open spectral dynamics (GKSL on a lattice). Discretize the quadratic Hamiltonian $H^{(2)}$ from Sec. 1.3 on a cubic mesh with spacing a and implement a cone–local GKSL generator:

$$\dot{\rho} \;=\; -i[H^{(2)},\rho] - \sum_{a,\mathbf{x}\in\Lambda} \gamma_a(\mathbf{x}) \left(L_{a,\mathbf{x}} \rho L_{a,\mathbf{x}}^\dagger - \tfrac{1}{2}\{L_{a,\mathbf{x}}^\dagger L_{a,\mathbf{x}}, \rho\} \right), \tag{1.121}$$

with $L_{a,\mathbf{x}}$ supported in a ball $B_{c_{\mathrm{eff}}\Delta t}(\mathbf{x})$ to preserve cone locality.

Schemes. Use Strang splitting (unitary + dissipative) per step Δt; verify CP by ensuring the discrete Kossakowski matrix is positive semidefinite at machine precision. For large systems, employ stochastic unraveling (quantum trajectories) with ensemble averaging.

Double–slit (phase–only tagging). Geometry: two slits with equal transmission, a thin phase plate producing $\delta\phi$ but negligible attenuation. Observable: fringe shift at fixed count rate. Model with a local dephaser $L_\phi \propto \int_{\mathrm{slit}} d^2x\, \Phi^\dagger\Phi$ and rate $\Gamma_\phi \sim \eta k^2/\kappa$; fit $\mathcal{V}(k,\theta)$ vs. $\delta\phi$ as in Sec. 1.7.1.

Colored Zeno (waveguide / cold atoms). Pulse trains with spectral weight $n_{\mathrm{pulse}}(\omega)$ define the colored Zeno parameter $\mathcal{Z}(\omega) = \mathcal{J}(\omega) n_{\mathrm{pulse}}(\omega)$. Scan across the crossover $\mathcal{Z}(\omega_0) \approx 1$ to observe Zeno $\to$ anti–Zeno transitions (Sec. 1.7.2).

Datasets. (i) Laboratory: cold–atom interferometry, photonic lattices, superconducting circuits for engineered dephasing; (ii) Cosmological tie–ins (validation of emergent isotropy): CMB g_* bounds on statistical anisotropy (angle–averaged templates), PTA/LIGO constraints on c_T (late–time restoration), to be used as *external priors* when calibrating simulator parameters.

1.9.3 Falsifiable signatures

We list crisp, near–term tests with quantitative targets.

(F1) Phase–only double–slit shift at fixed counts. Prediction: a *pure phase* tag near the slits shifts the interference pattern without reducing the integrated intensity when $\Gamma_\phi L/c_{\mathrm{eff}} \ll 1$ (no absorption channel). Quantitatively, for path difference $\Delta\ell$ and plate phase $\delta\phi$, the fringe maximum satisfies

$$k\,\Delta\ell + \delta\phi + \Delta\phi_{\mathrm{flood}} \;=\; 2\pi m, \qquad \Delta\phi_{\mathrm{flood}} \;=\; \int_{\mathrm{path}} \frac{\Gamma_\phi}{2\kappa}\, dt \;+\; \mathcal{O}(\epsilon_{\mathrm{aniso}}), \tag{1.122}$$

with visibility $\mathcal{V}$ unchanged at leading order. *Discriminator:* orthodox models with ideal phase plates predict no $\Delta\phi_{\mathrm{flood}}$ term.

(F2) Colored Zeno crossover. Let Γ_0 be the natural decay/dephasing rate and $\mathcal{Z}(\omega)$ the colored Zeno parameter. The survival probability $P(t)$ exhibits:

$$\text{Zeno:}\;\; \left.\frac{d\Gamma_{\mathrm{eff}}}{d\mathcal{Z}}\right|_{\omega_0} < 0, \qquad \text{anti–Zeno:}\;\; \left.\frac{d\Gamma_{\mathrm{eff}}}{d\mathcal{Z}}\right|_{\omega_0} > 0, \tag{1.123}$$

with crossover at $\mathcal{Z}(\omega_0) \approx 1$ where ω_0 matches the dominant system transition. *Discriminator:* rate vs. spectrum dependence, not merely vs. repetition time, fixes the "flooding" mechanism.

(F3) Entanglement thread–cut. Prepare bipartite entanglement with CHSH value β_0 close to $2\sqrt{2}$. Apply a *local* impairment to the χ^μ–thread in region $\mathcal{R}_A$, modeled by a dissipator $L_\chi \propto \nabla\cdot\chi$ with rate γ_χ. Prediction:

$$\beta(\gamma_\chi, L) \;=\; \beta_0\, e^{-\gamma_\chi \tau(L)} \;+\; \mathcal{O}(\epsilon_{\text{aniso}}), \qquad \left.\frac{\partial\langle O_B\rangle}{\partial\gamma_\chi}\right|_{\mathcal{R}_A} = 0, \tag{1.124}$$

where $\tau(L)$ depends on thread length L; no change in single–party marginals (no–signalling). *Discriminator:* correlation decay without marginal drift.

Outlook. The bridge in §1.9.1 motivates a first–principles derivation of gauge kinematics from fiber geometry; the protocols in §1.9.2 enable controlled laboratory tests of the spectral–quantum postulates; and the signatures in §1.9.3 offer clear pass/fail criteria. Together they chart a pragmatic path from ontological claims to quantitative confrontation with data.

1.10 Notation and Parameter Map (for quick reference)

This section compiles the core symbols, open–system channels, natural time/length scales, and a compact parameter→observable map used throughout Volume III. Cross–references point to the places where each quantity is defined or first constrained.

Core fields, operators, and geometric data.

Symbol	Meaning	Notes / References
$\mathcal{S}(x) = \{\Phi, \rho, \eta, \chi^\mu, T\}$	Spectral medium fields	Vol. I, Ch. 2; Secs. 1.2, 1.3
$g_{\text{eff}}[\mathcal{S}]$	Emergent effective metric	Vol. I, Ch. 2–3; cone for causality
u^μ	Aether/rest frame (unit timelike)	Vol. I, Ch. 2; used in coarse–graining
D_μ	Spectral covariant derivative	Vol. I, Ch. 2; cf. (1.118)
$\hat{\chi} \equiv \chi^\mu/\sqrt{\chi^2}$	Director (anisotropy axis)	Sets longitudinal/transverse split
$H^{(2)}$	Quadratic Hamiltonian (normal modes)	Sec. 1.3
$\mathsf{K}, \mathsf{C}, \mathsf{Z}^{ij}, \mathsf{M}^2$	Quadratic blocks in $\mathcal{L}^{(2)}$	Eq. (**??**)
$c_s(\theta)$	Scalar-mode sound speed (angle–dependent)	Secs. 1.3, **??** (Vol. II)
c_T	Tensor-mode speed	Vol. II, Sec. **??**; App. F
$M_{\text{eff}}(\rho)$	State–dependent Planck scale	Vol. I, Ch. 2; Vol. II, Sec. **??**

Core notation for fields, geometry, and quadratic dynamics.

Open–system (GKSL) data and channels.

Natural timescales, length scales, and thresholds.

Running parameters and consistency gates (quick check).

- *Running set (examples):* $\{Z_\perp,\ \Delta Z,\ c_s(\theta),\ c_T,\ M_{\text{eff}},\ \gamma_a\}$.
- *Gates for quantum/open sector (GQ):*

  GQ1 Causality Front velocities $\le c_{\text{eff}}$ (cone of g_{eff}).
  GQ2 CP Kossakowski matrix $\succeq 0 \Rightarrow$ complete positivity.

Object	Form / Example	Role / Reference
H_{eff}	$H^{(2)} + H_{\text{LS}}$ (Lamb–shift)	Generator of unitary part; Sec. 1.4
L_a (dephasing)	$L_\phi \propto \Phi^\dagger \Phi$ (localized)	Phase–only tagging; Secs. 1.4.3, 1.7.1
L_a (relaxation)	$L_- \propto \Phi$ or mode–lowering op.	Amplitude damping; Sec. 1.4
L_a (thread impairment)	$L_\chi \propto \nabla \cdot \chi$ (local)	Entanglement "thread–cut"; Sec. 1.6.3
$\gamma_a(\mathcal{S})$	State–dependent rates (e.g. Γ_ϕ)	CP/causal kernels; Sec. 1.4.3
$\mathcal{D}[\rho]$	$\sum_a \gamma_a (L_a \rho L_a^\dagger - \frac{1}{2}\{\cdot, \rho\})$	GKSL dissipator; Eq. (1.42)

Lindblad ingredients and prototypical channels.

Scale	Definition	Interpretation / Use
τ_{lock}	Phase–locking time of pointer modes	Measurement stabilization; Sec. 1.5.1
τ_ϕ	$\tau_\phi \equiv \Gamma_\phi^{-1}$	Dephasing time (phase–only tagging); Sec. 1.7.1
ℓ_ϕ	$\ell_\phi \equiv c_{\text{eff}} \tau_\phi$	Dephasing length within g_{eff} cone
τ_c	Env. correlation time	Markovianity bound; Sec. 1.4.2
ξ	Correlation length of $\mathcal{S}$	Coarse–graining cutoff; Vol. II, App. C
Λ_{sc}	Strong–coupling scale of EFT	Gate: $\Lambda_{\text{sc}} > \max(H, \omega)$; Sec. 1.8
$\mathcal{Z}(\omega)$	$\int d\omega\, \mathcal{J}(\omega) n_{\text{pulse}}(\omega)$	Colored Zeno parameter; Sec. 1.7.2

Operational scales and their roles.

GQ3 Entropy $\dot{S}_{\text{vN}} \geq 0$ for $\eta > 0$; Eq. (**??**).

GQ4 Restoration Isotropy/linearity in limits $\eta \downarrow 0$, coarse–grained IR.

- *Global gates (S/G/TQ/D):* No ghosts ($\mathsf{K} \succ 0$), hyperbolicity and subluminality ($c_s^2 > 0$, $c_T^2 > 0$, front$\leq c_{\text{eff}}$), topological consistency (anomaly cancellation or WZW/CS counterterms), and data gates (observational priors). See Sec. 1.8.

Parameter → observable map (indicative).

Parameter / Combination	Primary observable(s)	Where constrained
$c_s(\theta)$, M_{eff}, ϵ_η	$\mathcal{P}_\zeta(k, \theta)$, n_s	Vol. II, Sec. **??**
c_T, M_{eff}	$\mathcal{P}_T$, r, GW speeds	Vol. II, Sec. **??**; App. F
ϵ_{aniso}, $\hat{\chi}$	Statistical anisotropy g_*	Vol. II, Sec. **??**
(Wilson coeffs of $S^{(3)}$), $c_s(\theta)$	f_{NL} shapes (incl. directional)	Vol. II, App. B; V3 Sec. 1.7
Γ_ϕ (τ_ϕ), τ_{lock}	Double–slit fringe shift at fixed counts	Sec. 1.7.1
$\mathcal{J}(\omega)$, $n_{\text{pulse}}(\omega)$	Zeno ↔ anti–Zeno crossover	Sec. 1.7.2
γ_χ, thread length L	CHSH degradation without signalling	Sec. 1.6.3
Anomaly functional $\mathcal{A}_I$	Ward identities / consistency of $\mathcal{G}$	Sec. 1.9.1

Pointers from spectral parameters to measurable signatures.

Unit conventions. Unless stated otherwise, we work with $c = \hbar = 1$ and measure times/lengths with respect to g_{eff}; front velocities are always checked against the g_{eff} cone (GQ1).

Part II

Classical Recovery: Mass, Gravity, and Motion

Chapter 2

Mass, Gravity, and Worldline Dynamics

2.1 What is Mass? A Spectral Definition

Setup. Consider a (possibly localized) stationary configuration $\mathcal{S}_\star(x)$ of the spectral medium $\mathcal{S}(x) = \{\Phi, \rho, \eta, \chi^\mu, T\}$ (Vol. I, Ch. 2) solving the Euler–Lagrange equations of the unified action (Vol. I, Ch. 3). Around $\mathcal{S}_\star$, introduce collective coordinates (moduli) for symmetries:

$$\mathcal{S}(x;t) \;=\; \mathcal{S}_\star\Big(x - X(t);\; \hat{\chi}(t),\; \alpha(t),\; \dots\Big) \;+\; \delta\mathcal{S}(x;t),$$

where $X^i(t)$ is the center coordinate, $\hat{\chi}$ internal director orientation, and α a global phase (when applicable). After inserting in the action and integrating spatially, the leading quadratic (in velocities) effective Lagrangian takes the form

$$L_{\text{eff}} \;=\; -E_\star \;+\; \tfrac{1}{2}\, M_{ij}\, \dot{X}^i \dot{X}^j \;+\; \tfrac{1}{2}\, I_{ab}\, \Omega^a \Omega^b \;+\; \cdots, \tag{2.1}$$

where $E_\star$ is the rest energy of $\mathcal{S}_\star$, M_{ij} the *inertial mass matrix*, I_{ab} generalized moments for internal rotations (with angular velocities Ω^a), and dots denote higher multipoles and radiative terms (Vol. I, Ch. 6).

Inertial mass (spectral). Define the inertial mass tensor by the Hessian of the energy with respect to a uniform boost of the configuration on the emergent metric g_{eff}:

$$M_{ij} \;\equiv\; \left.\frac{\partial^2 E[\mathbf{v}]}{\partial v^i\, \partial v^j}\right|_{\mathbf{v}=0}, \qquad E[\mathbf{v}] \;=\; \int d^3x\, T^{00}_{\text{eff}}\Big[\Lambda(\mathbf{v})\cdot\mathcal{S}_\star\Big], \tag{2.2}$$

with $T^{\mu\nu}_{\text{eff}}$ the coarse-grained stress–energy built from D_μ and $g_{\text{eff}}[\mathcal{S}]$ (Vol. I, Ch. 3), and $\Lambda(\mathbf{v})$ the small-velocity boost in the aether frame u^μ. If anisotropies from $\hat{\chi}$ are negligible at the scale of $\mathcal{S}_\star$, then $M_{ij} \simeq M\,\delta_{ij}$ and we *define* the inertial mass

$$M \;\equiv\; \tfrac{1}{3}\,\mathrm{Tr}\, M_{ij}. \tag{2.3}$$

Spectral ADM-like mass and energy–momentum. Independently, define total energy and momentum in the aether rest frame

$$E \;=\; \int d^3x\, T^{00}_{\text{eff}}, \qquad P^i \;=\; \int d^3x\, T^{0i}_{\text{eff}}. \tag{2.4}$$

When an approximately Lorentzian $g_{\rm eff}$ holds (Vol. I, Sec. 2.5), one has the invariant

$$M_{\rm inv}^2 \;=\; E^2 - g_{ij}^{\rm eff}\, P^i P^j, \tag{2.5}$$

and in the nonrelativistic limit $E = E_\star + \frac{P^2}{2M} + \cdots$ the M extracted from (2.5) matches (2.3).

Gravitational mass (active/passive). Varying the action with respect to the emergent metric yields $T^{\mu\nu}_{\rm eff} \equiv \frac{2}{\sqrt{-g_{\rm eff}}}\frac{\delta S}{\delta g^{\rm eff}_{\mu\nu}}$. In the weak-field, slow-motion regime ($g_{00}^{\rm eff} = -1 - 2\Phi_N$, $g_{0i}^{\rm eff} \approx 0$, $g_{ij}^{\rm eff} \approx \delta_{ij}$), the spectral Palatini equations reduce to a Poisson law

$$\nabla^2 \Phi_N \;=\; 4\pi G_{\rm eff}\, \rho_{\rm energy}, \qquad G_{\rm eff} \;\equiv\; \frac{1}{8\pi\, M_{\rm eff}^2(\rho_0)}\,, \tag{2.6}$$

with $\rho_{\rm energy} \equiv T^{00}_{\rm eff}$ and $M_{\rm eff}(\rho_0)$ the state-dependent Planck scale (Vol. I, Sec. 2.6). The *active* gravitational mass sourcing Φ_N is $M^{(\rm act)}_{\rm grav} = \int d^3x\, \rho_{\rm energy}$; the *passive* mass is fixed by the universal minimal coupling of worldlines to $g_{\rm eff}$ (next section). Diffeomorphism invariance of the effective action implies equality,

$$M_{\rm inertial} \;=\; M^{(\rm passive)}_{\rm gravitational} \;=\; M^{(\rm active)}_{\rm gravitational}, \tag{2.7}$$

i.e. an emergent equivalence principle provided the gates of energy positivity and causal propagation hold (Vol. I, Gates U1–U3).

Remarks.

1. **Anisotropy and mass tensor.** If the director $\hat{\chi}$ is long-range ordered, M_{ij} can be anisotropic. Observational constraints (solar-system and binary pulsars) bound $|M_\parallel - M_\perp|/M \ll 1$, constraining spectral anisotropy at macroscopic scales.

2. **Dissipation η and mass renormalization.** A small spectral viscosity $\eta > 0$ introduces Rayleigh terms that *renormalize* M_{ij} at finite frequency. In the static limit, the gate $\sigma_{\rm diss} \geq 0$ ensures no negative-mass pathologies.

3. **Composite objects.** For multi-soliton bound states, M adds with a binding correction from the inter-soliton potential $U_{\rm ad}$ (Vol. I, Ch. 7): $M_{\rm bound} = \sum_a M_a + U_{\rm ad}/c^2_{\rm eff} + \cdots$.

In what follows we show that geodesic motion in $g_{\rm eff}$ together with well-controlled multipole and spin couplings reproduces Newtonian and post-Newtonian laws, thereby anchoring classical dynamics in spectral first principles.

2.2 Emergent Gravity: Field Equations and Weak-Field Limit

Palatini–spectral field equations. Recall (Vol. I, Ch. 3) the unified action in a Palatini-like form where the coframe/connection are built from the structural fields:

$$S_{\rm tot}[\mathcal{S}] = \int d^4x\, \sqrt{-g_{\rm eff}[\mathcal{S}]}\, \left\{ \frac{M^2_{\rm eff}(\rho)}{2}\, \mathcal{R}[g_{\rm eff}, \omega(\mathcal{S})] + \mathcal{L}_{\rm struct}[\Phi, \rho, \eta, \chi^\mu, T; D_\mu] + \mathcal{L}^{\rm eff}_{\rm mat} \right\}. \tag{2.8}$$

Variation w.r.t. $g^{\rm eff}_{\mu\nu}$ gives

$$M^2_{\rm eff}(\rho)\, G_{\mu\nu}[g_{\rm eff}] = \; T^{(\rm struct)}_{\mu\nu} + T^{(\rm mat)}_{\mu\nu} \; + \; \Theta_{\mu\nu}[\mathcal{S}], \tag{2.9}$$

where $\Theta_{\mu\nu}$ collects non-EH curvature responses (higher gradients and mild anisotropies induced by $\hat{\chi}$). Energy–momentum is defined by $T^{\rm eff}_{\mu\nu} = \frac{2}{\sqrt{-g_{\rm eff}}}\frac{\delta S}{\delta g^{\mu\nu}_{\rm eff}}$.

Aether frame and linearization. Linearize around a homogeneous ordered state S_0 with aether 4-velocity u^μ and nearly Minkowskian $g^{(0)}_{\mu\nu} = \eta_{\mu\nu}$:

$$g^{\rm eff}_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}, \qquad |h_{\mu\nu}| \ll 1, \qquad u^\mu = (1, \mathbf{0}) + O(h). \tag{2.10}$$

In generalized de Donder gauge, the linear equations reduce to

$$\Box \bar{h}_{\mu\nu} = -\frac{2}{M^2_{\rm eff}(\rho_0)}\, T^{\rm eff}_{\mu\nu} \; + \; \mathcal{A}_{\mu\nu}[\hat{\chi}, \partial]\,, \tag{2.11}$$

where $\bar{h}_{\mu\nu} = h_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu}h$, and $\mathcal{A}_{\mu\nu}$ encodes suppressed anisotropic operators (e.g. $(\hat{\chi}\cdot\nabla)^2$ terms) consistent with the gates.

Newtonian limit and Poisson law. In the slow-motion, weak-field, quasi-static regime, $h_{00} = -2\Phi_N$, $h_{0i} \simeq 0$, $h_{ij} \simeq 2\Psi\,\delta_{ij}$. Neglecting $\mathcal{A}_{\mu\nu}$ at leading order one finds

$$\nabla^2 \Phi_N = 4\pi G_{\rm eff}\, \rho_{\rm energy}\,, \qquad G_{\rm eff} \equiv \frac{1}{8\pi\, M^2_{\rm eff}(\rho_0)}\,, \tag{2.12}$$

with $\rho_{\rm energy} = T^{00}_{\rm eff}$. Anisotropic stress from the structural sector can produce $\Phi_N - \Psi = O(\epsilon_\chi)$; solar-system bounds impose $|\Phi_N - \Psi|/\Phi_N \ll 10^{-5}$, hence $\epsilon_\chi \ll 10^{-5}$ on macroscales.

2.3 Worldline Principle and Geodesic Motion

From $\nabla_\mu T^{\mu\nu} = 0$ to an effective worldline. For a localized solitonic configuration $\mathcal{S}_\star$, define the total momentum on a Cauchy 3-surface Σ_τ

$$P^\nu(\tau) = \int_{\Sigma_\tau} d\Sigma_\mu\, T^{\mu\nu}_{\rm eff}\,, \qquad \frac{DP^\nu}{d\tau} = 0 + O(\text{radiative}), \tag{2.13}$$

where $D/d\tau = u^\mu \nabla_\mu$ along a representative center-of-energy curve $z^\mu(\tau)$. At *monopole order* (neglecting spin/multipoles) one can show $P^\mu = M u^\mu$ and

$$u^\alpha \nabla_\alpha u^\mu = 0 \quad \Longleftrightarrow \quad \frac{d^2 z^\mu}{d\tau^2} + \Gamma^\mu_{\alpha\beta}(g_{\rm eff})\frac{dz^\alpha}{d\tau}\frac{dz^\beta}{d\tau} = 0\,, \tag{2.14}$$

i.e. free solitons follow $g_{\rm eff}$ geodesics.

Worldline action and variation. Equivalently, start from the minimal worldline action

$$S_{\rm wl}[z^\mu] = -M \int d\tau\,, \qquad d\tau^2 = -g^{\rm eff}_{\mu\nu}\, dz^\mu dz^\nu, \tag{2.15}$$

whose Euler–Lagrange equations reproduce (4.1). Small Rayleigh dissipation inherited from $\eta > 0$ can be incorporated via $\Delta S_{\rm wl} = -\frac{1}{2}\int d\tau\, \gamma_{\rm wl}\, g^{\rm eff}_{\mu\nu} u^\mu u^\nu$ with $\gamma_{\rm wl} \geq 0$; this shifts $DP^\mu/d\tau$ by $O(\gamma_{\rm wl} u^\mu)$ and is negligible under the gate $\sigma_{\rm diss} \geq 0$ at astrophysical scales.

Newtonian limit. With $g_{00}^{\rm eff} = -1 - 2\Phi_N$, $g_{ij}^{\rm eff} = (1-2\Psi)\delta_{ij}$ and $|\mathbf{v}| \ll 1$,

$$\frac{d^2\mathbf{X}}{dt^2} = -\nabla\Phi_N(\mathbf{X}) \; + \; O(v^2, \Phi_N^2, \epsilon_\chi)\,, \tag{2.16}$$

thereby recovering Newton's law with $G_{\rm eff}$ from (2.12).

2.4 Finite-Size Structure: Spin and Multipoles (MPD System)

Moments and center conditions. Define the spin tensor and higher moments (Dixon):

$$S^{\mu\nu}(\tau) = \int_{\Sigma_\tau} d\Sigma_\alpha \, [(x^\mu - z^\mu)T_{\rm eff}^{\alpha\nu} - (x^\nu - z^\nu)T_{\rm eff}^{\alpha\mu}]\,, \qquad J^{\alpha\beta\gamma\delta} = \int_{\Sigma_\tau} d\Sigma_\lambda \, (x-z)^{[\alpha}(x-z)^{\beta]}(x-z)^{[\gamma}T_{\rm eff}^{\delta]\lambda}, \tag{2.17}$$

and choose a spin supplementary condition (SSC), e.g. Tulczyjew $S^{\mu\nu}P_\nu = 0$.

MPD equations on $g_{\rm eff}$. Up to quadrupole order one finds

$$\frac{DP^\mu}{d\tau} = -\frac{1}{2}\, R^\mu{}_{\nu\alpha\beta}[g_{\rm eff}]\, u^\nu S^{\alpha\beta} - \frac{1}{6}\, J^{\alpha\beta\gamma\delta}\, \nabla^\mu R_{\alpha\beta\gamma\delta} + O(\text{higher})\,, \tag{2.18}$$

$$\frac{DS^{\mu\nu}}{d\tau} = 2P^{[\mu}u^{\nu]} + O(\text{quadrupole})\,, \tag{2.19}$$

where $P^\mu = Mu^\mu + O(S^2, J)$. These equations capture spin–curvature coupling (Lense–Thirring precession) and tidal forces via $J^{\alpha\beta\gamma\delta}$.

Spectral anisotropy couplings (suppressed). The internal director $\hat{\chi}$ allows symmetry-permitted but constrained couplings (all suppressed by gates and data), e.g.

$$\Delta\mathcal{L}_{\rm spin-\chi} = \frac{\kappa_\chi S}{2M}\, S^{\mu\nu}\,(\nabla_\mu\hat{\chi}_\nu - \nabla_\nu\hat{\chi}_\mu) + \frac{\lambda_\chi}{4}\, J^{\alpha\beta\gamma\delta}\hat{\chi}_\alpha\hat{\chi}_\gamma\, R_{\beta\delta} + \cdots\,, \tag{2.20}$$

which are bounded by PPN and binary-pulsar tests to be $|\kappa_\chi S|, |\lambda_\chi| \ll 1$ on macroscales.

2.5 Post-Newtonian (PN) Recovery and PPN Parameters

1PN two-body Lagrangian. For two solitonic bodies A, B with masses M_A, M_B, relative position $\mathbf{r} = \mathbf{X}_A - \mathbf{X}_B$, $r = |\mathbf{r}|$, and velocities $\mathbf{v}_{A,B}$, the conservative 1PN Lagrangian in the isotropic limit reads

$$L_{\rm 1PN} = \sum_A \frac{M_A v_A^2}{2} + \frac{G_{\rm eff} M_A M_B}{r} + \frac{1}{c_{\rm eff}^2}\Bigg[\sum_A \frac{M_A v_A^4}{8} + \frac{G_{\rm eff} M_A M_B}{2r}\Big(3(v_A^2 + v_B^2) - 7\,\mathbf{v}_A\cdot\mathbf{v}_B - \frac{(\mathbf{r}\cdot\mathbf{v}_A)(\mathbf{r}\cdot\mathbf{v}_B)}{r^2}$$
$$- \frac{G_{\rm eff}^2 M_A M_B (M_A + M_B)}{2r^2}\Bigg] + O(c_{\rm eff}^{-4})\,, \tag{2.21}$$

where $c_{\rm eff}$ is the tensor-mode speed ($c_T \simeq 1$). Directional corrections from $\hat{\chi}$ appear as $O(\epsilon_\chi)$ re-weightings of velocity and potential terms and are constrained $\ll 10^{-5}$.

PPN parameters. Comparing the metric solution to the standard PPN form yields

$$\gamma = 1 + \delta\gamma(\epsilon_\chi, \Theta_{\mu\nu}), \qquad \beta = 1 + \delta\beta(\partial M_{\rm eff}, \Theta_{\mu\nu}), \qquad \alpha_1, \alpha_2 = O(\epsilon_\chi)\,, \tag{2.22}$$

with Cassini and pulsar bounds implying $|\delta\gamma| \lesssim 2\times10^{-5}$, $|\delta\beta| \lesssim 10^{-4}$, and $|\alpha_{1,2}| \lesssim 10^{-5}$. These map to tight limits on long-range anisotropy and on gradients of $M_{\rm eff}(\rho)$ in the solar neighborhood.

2.6 Dissipative Corrections and Radiation Reaction

Spectral Rayleigh drag (negligible astrophysically). A small but positive viscosity $\eta > 0$ yields a Rayleigh functional $\mathcal{R} = \frac{1}{2}\int d^3x\,\Gamma(\mathcal{S})\,(\partial_t\mathcal{S})^2$. For a compact body moving slowly through a nearly homogeneous background this induces

$$\mathbf{F}_{\rm diss} = -\gamma_{\rm cm}\,\mathbf{v}\,, \qquad \gamma_{\rm cm} \sim \int d^3x\,\Gamma(\mathcal{S}_0)\,|\nabla\mathcal{S}_\star|^2, \tag{2.23}$$

which is constrained by planetary ephemerides to be $\gamma_{\rm cm} \approx 0$ on solar-system scales (gate $\sigma_{\rm diss} \geq 0$ with tiny coefficient).

Gravitational radiation and c_T. Tensor waves propagate with speed $c_T = 1 + \delta c_T(\mathcal{S})$ (Vol. II, Sec. **??**). The quadrupole power is

$$P_{\rm GW} = \frac{G_{\rm eff}}{5c_T^5}\left\langle \dddot{Q}_{ij}\,\dddot{Q}_{ij}\right\rangle [1 + O(\epsilon_\chi)]\,, \tag{2.24}$$

with $|\delta c_T| \ll 10^{-15}$ from multi-messenger events. Standard radiation-reaction terms (Burke–Thorne at 2.5PN) follow with $G \to G_{\rm eff}$, $c \to c_T$ and are fully compatible with binary-pulsar and LIGO/Virgo/KAGRA observations within current bounds.

2.7 Synthesis: Equivalence Principle and Classical Closure

Equality of masses and universality of free fall. Within the gates (no ghosts, causal cones within $g_{\rm eff}$, nonnegative entropy production), the same $T^{\rm eff}_{\mu\nu}$ sources the metric (2.9) and is transported by $\nabla_\mu T^{\mu\nu} = 0$, implying

$$M_{\rm inertial} = M_{\rm grav}^{\rm (active)} = M_{\rm grav}^{\rm (passive)}\,, \tag{2.25}$$

hence universal geodesic motion at monopole order (EEP). Finite-size/anisotropy corrections enter only through controlled, testable MPD terms (2.18)–(2.19) and are observationally bounded.

Classical limit of the spectral program. We have shown that the spectral definitions of mass, stress–energy, and emergent geometry reproduce Newtonian dynamics, PN corrections, spin dynamics, and radiation reaction, all within the same pre-geometric medium and its gates. This anchors the *classical recovery* and prepares the ground to connect these worldline sectors with the quantum (spectral) sectors in the next chapters.

Table 2.1: PPN ↔ Spectral map (leading order) and indicative bounds. Small anisotropy is encoded by $\epsilon_\chi \equiv |c_\parallel^2 - c_\perp^2|$ and non-Einstein curvature responses by $\Theta_{\mu\nu}$. Numbers are representative (solar-system & pulsar).

PPN param.	**Spectral origin**	**Leading expression**	**Typical bou**
$\gamma - 1$	Anisotropic stress / higher-curv.	$\gamma - 1 \simeq \Pi_1[\Theta_{\mu\nu}, \epsilon_\chi]$	$\lesssim 2 \times 10^{-5}$
$\beta - 1$	State-dep. Planck mass	$\beta - 1 \simeq \Pi_2[\partial_\rho \ln M_{\rm eff}]$	$\lesssim 10^{-4}$
α_1, α_2	Preferred-frame via $\hat{\chi}$	$\alpha_{1,2} \sim \mathcal{O}(\epsilon_\chi)$	$\lesssim 10^{-5}$
G variation	Running $M_{\rm eff}(\rho, t)$	$\dfrac{\dot{G}_{\rm eff}}{G_{\rm eff}} = -2\dfrac{\dot{M}_{\rm eff}}{M_{\rm eff}}$	$\lesssim 10^{-13}$ yr^{-1}
Light deflection	γ (and c_T)	$\Delta\theta = \dfrac{1+\gamma}{2}\dfrac{4G_{\rm eff}M}{b\,c_T^2}$	$\sim 10^{-4}$ rel. ac
Shapiro delay	γ (and c_T)	$\Delta t \propto (1+\gamma)\,\ln\dfrac{r_S + r_R + R}{r_S + r_R - R}$	$\sim 10^{-5}$ rel. ac
Perihelion advance	β, γ (and c_T)	$\Delta\varpi = \dfrac{6\pi G_{\rm eff}M}{a(1-e^2)c_T^2}\,\dfrac{2-\beta+2\gamma}{3}$	$\sim 10^{-3}$ rel. ac
GW speed	Tensor sector	$c_T = 1 + \delta c_T(\mathcal{S})$	$\lvert\delta c_T\rvert \lesssim 10^{-15}$

2.8 PPN–Spectral Parameter Map and Observational Bounds

Purpose. We summarize how the post-Newtonian (PPN) parameters map to spectral couplings/structures and list representative bounds that constrain the allowed region of the spectral model space (gates S/G/TQ/D).

Remarks. (i) $\Pi_{1,2}[\cdot]$ are linear functionals of the indicated small quantities, computable once the quadratic curvature sector (Vol. I, Ch. 3) is fixed. (ii) Bounds translate to constraints on $\{\epsilon_\chi,\ \Theta_{\mu\nu},\ \partial_\rho \ln M_{\rm eff}\}$, hence on the coefficients of the spectral Lagrangian controlling anisotropic stresses and the running of $M_{\rm eff}$ across ambient densities.

Gate view. In the PPN regime, the S/G/TQ/D gates imply:

(S) $\mathsf{K} \succ 0$, (G) $c_s^2, c_T^2 > 0$, (D) $v_{\rm front} \le c_T$, (Restoration) $\epsilon_\chi \to 0,\ \delta c_T \to 0$ on solar scales.

2.8.1 Light bending, Shapiro delay, and perihelion: closed forms

For a static, spherically-symmetric source of mass M,

$$\Delta\theta_{\rm bend} = \frac{1+\gamma}{2}\,\frac{4G_{\rm eff}M}{b\,c_T^2}\,[1 + O(\epsilon_\chi)], \tag{2.26}$$

$$\Delta t_{\rm Shapiro} = (1+\gamma)\,\frac{G_{\rm eff}M}{c_T^3}\,\ln\!\left(\frac{r_S + r_R + R}{r_S + r_R - R}\right)[1 + O(\epsilon_\chi)], \tag{2.27}$$

$$\Delta\varpi_{\rm orbit} = \frac{6\pi G_{\rm eff}M}{a(1-e^2)c_T^2}\,\frac{2-\beta+2\gamma}{3}\,[1 + O(\epsilon_\chi)], \tag{2.28}$$

with the usual notation for impact parameter b, semi-major axis a, and eccentricity e. Eqs. (2.26)–(4.6) reduce to GR when $G_{\rm eff} \to G$, $c_T \to 1$, $\beta, \gamma \to 1$.

2.9 Worked Example: Lense–Thirring Precession (Earth–LAGEOS)

Frame dragging in the spectral setting. For a spinning central body with angular momentum J and a circular orbiter at radius r, the nodal precession (Lense–Thirring) reads

$$\Omega_{\rm LT}^{(\rm spec)} = \frac{2\,G_{\rm eff}\,J}{c_T^2\,r^3}\Big[1+O(\epsilon_\chi)\Big]. \tag{2.29}$$

Using $G_{\rm eff} = 1/(8\pi M_{\rm eff}^2)$ and $c_T = 1+\delta c_T$, deviations scale as $\propto (G_{\rm eff}/G)\,(1-2\delta c_T)$ at leading order.

Numerical illustration (Earth–LAGEOS). Take $J_\oplus \simeq 5.86\times10^{33}\ \mathrm{kg\,m^2\,s^{-1}}$, $r \simeq 1.22\times10^7$ m. Then

$$\Omega_{\rm LT}^{(\rm GR)} = \frac{2GJ_\oplus}{c^2 r^3} \;\approx\; 4.8\times10^{-15}\ \mathrm{rad\,s^{-1}} \;\approx\; 31\ \mathrm{mas\,yr^{-1}}, \tag{2.30}$$

$$\Omega_{\rm LT}^{(\rm spec)} \approx \Omega_{\rm LT}^{(\rm GR)} \times \left(\frac{G_{\rm eff}}{G}\right)\left(\frac{1}{1+\delta c_T}\right)^2 [1+O(\epsilon_\chi)]. \tag{2.31}$$

Given $|\delta c_T| \lesssim 10^{-15}$ (multi-messenger) and $|G_{\rm eff}/G - 1| \ll 10^{-3}$ (ephemerides), the predicted shift is well below current LAGEOS error bars ($\sim$ 5–10%), enforcing tight bounds on long-range running of $M_{\rm eff}$ and on any preferred-frame contamination in the LT sector.

Spin–curvature and preferred-frame systematics. MPD corrections (2.18)–(2.19) introduce tiny latitude-dependent shifts $\propto \epsilon_\chi\,(\hat\chi\cdot\hat J)^2$ which can be searched for by comparing counter-rotating orbital planes; non-detection sets $\epsilon_\chi \lesssim 10^{-5}$ on Earth–satellite scales.

Outcome. The LT example illustrates how a single observable constrains the trio $\{G_{\rm eff}, c_T, \epsilon_\chi\}$ in a largely orthogonal way to the traditional light-propagation tests, thereby completing the classical consistency closure of the spectral model on Solar-System scales.

2.10 Emergent Electrodynamics from Spectral Phase

2.10.1 Spectral U(1) and field strength

The structural phase of the medium, Φ, admits a global shift $\Phi \to \Phi + \alpha$. Promoting this to a local redundancy selects an emergent spectral gauge potential A_μ via the minimal replacement

$$D_\mu\Phi \;\longrightarrow\; \mathcal{D}_\mu\Phi \equiv D_\mu\Phi - q\,A_\mu,$$

with q the spectral U(1) charge carried by phase excitations/solitons. The U(1) curvature is

$$F_{\mu\nu} \;:=\; \partial_\mu A_\nu - \partial_\nu A_\mu \qquad \text{(Abelian sector; non-Abelian generalizations in Vol. I, Sec. 6.14).}$$

At the effective level we allow anisotropic constitutive response of the medium encoded by $\varepsilon^{\mu\nu}$ and $\mu^{-1\,\mu\nu}$ built from $(g_{\rm eff}, \chi^\mu, \rho, \eta)$.

2.10.2 Effective Maxwell equations and constitutive relations

A gauge-invariant quadratic sector reads

$$\mathcal{L}_{\rm EM} = -\tfrac{1}{4}\,\mathsf{Z}_F^{\alpha\beta\gamma\delta}\,F_{\alpha\beta}F_{\gamma\delta} \;-\; J^\mu A_\mu, \qquad \mathsf{Z}_F^{\alpha\beta\gamma\delta} = \tfrac{1}{2}\Big(\varepsilon^{\alpha\gamma}g_{\rm eff}^{\beta\delta} - \varepsilon^{\alpha\delta}g_{\rm eff}^{\beta\gamma} - \varepsilon^{\beta\gamma}g_{\rm eff}^{\alpha\delta} + \varepsilon^{\beta\delta}g_{\rm eff}^{\alpha\gamma}\Big), \tag{2.32}$$

with four-current J^μ obtained by Noether from the phase symmetry of matter/soliton fields. Variation gives generalized Maxwell equations

$$\nabla_\mu\Big(\varepsilon^{\mu\alpha}F_\alpha{}^\nu\Big) = J^\nu, \qquad \nabla_{[\alpha}F_{\beta\gamma]} = 0. \tag{2.33}$$

In the aether frame u^μ define $\mathbf{E}_i = F_{i0}$, $\mathbf{B}^i = \tfrac{1}{2}\epsilon^{ijk}F_{jk}$, $\mathbf{D}_i = \varepsilon_{ij}\mathbf{E}_j$, $\mathbf{H}^i = \mu^{-1\,ij}\mathbf{B}_j$. Then (2.33) reduce to the macroscopic set

$$\begin{aligned} &\nabla\!\cdot\!\mathbf{D} = \rho_{\rm e}, && \nabla\!\cdot\!\mathbf{B} = 0,\\ &\nabla\!\times\!\mathbf{H} - \partial_t\mathbf{D} = \mathbf{J}, && \nabla\!\times\!\mathbf{E} + \partial_t\mathbf{B} = 0, \end{aligned}$$

with $\rho_{\rm e} = J^0$, $\mathbf{J}^i = J^i$. Anisotropy enters through $\varepsilon_{ij}(\rho,\eta,\hat\chi)$, $\mu_{ij}^{-1}(\rho,\eta,\hat\chi)$, generically diagonal in the $(\parallel,\perp)$ basis to $\hat\chi$.

2.10.3 Charges and currents from solitons

Let Ψ denote a charged soliton/matter field with spectral $U(1)$ phase. The Noether current is

$$J^\mu \equiv q\;\mathrm{Im}\Big(\Psi^*\,\mathcal{D}^\mu\Psi\Big) = \frac{q}{2i}\Big(\Psi^*\mathcal{D}^\mu\Psi - (\mathcal{D}^\mu\Psi)^*\Psi\Big). \tag{2.34}$$

up to small dissipative corrections sourced by $\eta > 0$ (accounted for via GKSL, Sec. 1.4).

2.10.4 Lorentz force and energy–momentum exchange

The equation of motion for a compact charged soliton with worldline $X^\mu(\tau)$ and effective mass $m_{\rm eff}$ reads

$$m_{\rm eff}\,u^\nu\nabla_\nu u^\mu \;=\; q\,F^\mu{}_\nu\,u^\nu \;+\; f^\mu_{\rm drag}[\eta] \;+\; \mathcal{O}(\nabla\varepsilon,\nabla\mu^{-1}), \tag{2.35}$$

with $u^\mu = dX^\mu/d\tau$ and $f^\mu_{\rm drag}$ a small GKSL-consistent damping force. Total energy–momentum conservation follows from $\nabla_\mu(T^{\mu\nu}_{\rm matter}+T^{\mu\nu}_{\rm EM}) = 0$, where $T^{\mu\nu}_{\rm EM} = \varepsilon^{\mu\alpha}F_{\alpha\beta}F^{\beta\nu} - \tfrac{1}{4}g^{\mu\nu}_{\rm eff}\varepsilon^{\alpha\gamma}F_{\alpha\beta}F_\gamma{}^\beta$.

2.10.5 Wave propagation, optics, and Poynting theorem

Plane waves $\propto e^{i(k\cdot x)}$ satisfy the generalized Fresnel equation, yielding direction-dependent phase velocities $c_{\parallel,\rm EM}, c_{\perp,\rm EM} \le 1$. The Poynting vector and energy density read

$$\mathbf{S} = \mathbf{E}\times\mathbf{H}, \qquad u_{\rm EM} = \tfrac{1}{2}(\mathbf{E}\!\cdot\!\mathbf{D} + \mathbf{B}\!\cdot\!\mathbf{H}),$$

and obey $\partial_t u_{\rm EM} + \nabla\!\cdot\!\mathbf{S} = -\mathbf{J}\!\cdot\!\mathbf{E}$.

2.10.6 Static limits and recovery of Coulomb/Biot–Savart

In stationary, homogeneous media ($\partial_t = 0$, constant ε_{ij}), $\nabla\!\cdot(\varepsilon\mathbf{E}) = \rho_{\rm e}$, $\nabla\!\times\!\mathbf{H} = \mathbf{J}$. For isotropic $\varepsilon = \varepsilon_\star$, $\mu = \mu_\star$ one recovers

$$\phi(\mathbf{x}) = \frac{1}{4\pi\varepsilon_\star}\int d^3x'\,\frac{\rho_{\rm e}(\mathbf{x}')}{|\mathbf{x}-\mathbf{x}'|}, \qquad \mathbf{B}(\mathbf{x}) = \frac{\mu_\star}{4\pi}\int d^3x'\,\frac{\mathbf{J}(\mathbf{x}')\times(\mathbf{x}-\mathbf{x}')}{|\mathbf{x}-\mathbf{x}'|^3}.$$

2.10.7 Radiation and reaction (GKSL-consistent)

Accelerated charges radiate power

$$P_{\rm rad} \;=\; \frac{q^2\,a^2}{6\pi}\,c_{\rm EM}^{-3} \quad \text{(isotropic limit)}, \tag{2.36}$$

with $c_{\rm EM} = 1/\sqrt{\varepsilon_\star \mu_\star}$. At the particle level we use a reduction-of-order Abraham–Lorentz–Dirac term regularized by the open-system channel (Sec. 1.4), which replaces pathological runaways by a small, causal friction consistent with entropy production $\sigma_{\rm diss} \geq 0$.

Consistency gates. No-ghost and hyperbolicity require $\varepsilon^{00} > 0$, positive-definite spatial ε_{ij}, μ_{ij}^{-1}, and front speeds within the $g_{\rm eff}$ cone. Restoration gate demands $c_{\parallel,\rm EM} - c_{\perp,\rm EM} \to 0$ on lab/solar scales.

2.11 Conservation Laws and Continuity Equations

We derive conservation/balance laws for energy–momentum, angular momentum, and spectral charges in the Quantum–Aether, starting from the unified action of Vol. I and allowing for open-system (GKSL) coarse–graining (Vol. III, Sec. 1.4). Throughout we work with the emergent metric $g_{\rm eff}$, aether frame u^μ, and projector $\Delta^{\mu\nu} \equiv g_{\rm eff}^{\mu\nu} + u^\mu u^\nu$.

2.11.1 Noether data and covariant balances

Stress-energy and diffeomorphism invariance. For the closed theory (no coarse–graining), diffeomorphism invariance yields the covariantly conserved stress–energy tensor

$$T^{\mu\nu} \;:=\; \frac{2}{\sqrt{-g_{\rm eff}}}\,\frac{\delta S_{\rm total}}{\delta g^{\rm eff}_{\mu\nu}}, \qquad \nabla_\mu T^{\mu\nu} = 0. \tag{2.37}$$

After tracing out fast/environmental modes (GKSL reduction), the *reduced* sector obeys a balance law

$$\nabla_\mu T^{\mu\nu}_{\rm red} \;=\; Q^\nu[\rho_{\rm red}], \qquad u_\nu\, Q^\nu \;\leq\; 0, \tag{2.38}$$

where the source Q^ν encodes dissipative channels and satisfies the entropy gate (Sec. 1.4). Total (system+environment) conservation remains exact.

Internal symmetries and currents. For any global internal symmetry (e.g. spectral U(1) of Φ) with matter/soliton field Ψ, Noether's theorem gives

$$J^\mu \;=\; \frac{\partial \mathcal{L}}{\partial(\nabla_\mu \Psi)}\,\delta\Psi \quad \Rightarrow \quad \nabla_\mu J^\mu \;=\; 0 \;\; \text{(closed)}, \qquad \nabla_\mu J^\mu \;=\; -\,\Gamma_{\rm leak} \;\; \text{(open)}. \tag{2.39}$$

For the emergent electrodynamics of Sec. 2.20, J^μ is given by Eq. (2.34) and is exactly conserved in the absence of leakage.

2.11.2 3+1 split: energy and momentum balances

Decompose $T^{\mu\nu}$ with respect to u^μ as

$$T^{\mu\nu} \;=\; \varepsilon\, u^\mu u^\nu + p\,\Delta^{\mu\nu} + u^{(\mu} q^{\nu)} + \pi^{\mu\nu}, \quad q^\mu u_\mu = 0, \quad \pi^\mu{}_\mu = 0, \quad \pi^{\mu\nu} u_\nu = 0, \tag{2.40}$$

where ε is the energy density, p the (isotropic) pressure, q^μ the heat/energy flux, and $\pi^{\mu\nu}$ the shear stress. Projecting Eq. (2.38) along u_ν and orthogonally gives

$$\text{Energy:} \quad u_\nu \nabla_\mu T^{\mu\nu} \;=\; -\dot\varepsilon - (\varepsilon + p)\,\theta - \nabla_\mu q^\mu - \pi^{\mu\nu}\sigma_{\mu\nu} \;=\; u_\nu Q^\nu, \tag{2.41}$$

$$\text{Momentum:} \quad \Delta^\alpha{}_\nu\, \nabla_\mu T^{\mu\nu} \;=\; (\varepsilon + p)\, a^\alpha + \nabla^\alpha p + \Delta^\alpha{}_\nu\,(\dot q^\nu + \theta\, q^\nu + \nabla_\mu \pi^{\mu\nu}) \;=\; \Delta^\alpha{}_\nu Q^\nu, \tag{2.42}$$

with kinematic scalars/tensors $\theta \equiv \nabla_\mu u^\mu$, $a^\alpha \equiv u^\mu \nabla_\mu u^\alpha$, $\sigma_{\mu\nu} \equiv \nabla_{\langle\mu} u_{\nu\rangle}$ (the symmetric, traceless, transverse part).

Constitutive relations (first order). To close Eqs. (2.41)–(2.42), we use first-order, GKSL-consistent constitutive laws

$$q^\mu \;=\; -\,\kappa_T(\mathcal{S})\,\Delta^{\mu\nu}\nabla_\nu T, \qquad \pi^{\mu\nu} \;=\; -\,2\,\hat\eta(\mathcal{S})\,\sigma^{\mu\nu} - \zeta(\mathcal{S})\,\theta\,\Delta^{\mu\nu}, \tag{2.43}$$

where the transport coefficients are *functions* of the spectral state: $\hat\eta, \zeta, \kappa_T = \hat\eta(\rho, \eta, \chi^\mu, T)$. (The symbol $\hat\eta$ denotes the *shear viscosity coefficient* to avoid collision with the spectral field η.)

2.11.3 Angular momentum and Belinfante symmetrization

Define the total (orbital+spin) angular-momentum current

$$J^{\lambda\mu\nu} \;=\; x^\mu T^{\lambda\nu} - x^\nu T^{\lambda\mu} + S^{\lambda\mu\nu}, \qquad S^{\lambda\mu\nu} \;=\; \text{spin current}. \tag{2.44}$$

In the Belinfante-improved scheme with symmetric $T^{\mu\nu}$, $\nabla_\lambda J^{\lambda\mu\nu} = 0$ follows from $\nabla_\mu T^{\mu\nu} = 0$ (closed theory). In open settings, $\nabla_\lambda J^{\lambda\mu\nu} = (x^\mu Q^\nu - x^\nu Q^\mu) + \nabla_\lambda S^{\lambda\mu\nu}$.

2.11.4 Charge, mass/number, and phase continuity

Spectral U(1) charge. With emergent $U(1)$ of Sec. 2.20,

$$\nabla_\mu J^\mu \;=\; 0 \quad \text{(exact in closed)}, \qquad \nabla_\mu J^\mu \;=\; -\,\Gamma_{\text{leak}}(\mathcal{S}) \le 0 \quad \text{(open)}. \tag{2.45}$$

Spectral mass/number. Introduce a mass/number current associated with the density field ρ,

$$N^\mu \;\equiv\; \rho\, u^\mu + \nu^\mu, \qquad \nu^\mu u_\mu = 0, \tag{2.46}$$

with Fick-type diffusion $\nu^\mu = -D_\rho(\mathcal{S})\,\Delta^{\mu\nu}\nabla_\nu \rho$. Balance takes the form

$$\nabla_\mu N^\mu \;=\; \Sigma_\rho(\mathcal{S}), \tag{2.47}$$

where Σ_ρ describes internal conversion between sectors; for a closed, single-sector flow $\Sigma_\rho = 0$.

2.11.5 Entropy current and the arrow of time

Define the entropy current (Landau frame)

$$s^\mu \;=\; s\,u^\mu - \frac{1}{T}\,q^\mu - \frac{\mu}{T}\,\nu^\mu, \tag{2.48}$$

with chemical potential μ for J^μ. Using Eqs. (2.41)–(2.43) and the Gibbs relation gives

$$\nabla_\mu s^\mu \;=\; \frac{\hat\eta}{T}\,\sigma_{\mu\nu}\sigma^{\mu\nu} + \frac{\zeta}{T}\,\theta^2 + \frac{\kappa_T}{T^2}\,\Delta^{\mu\nu}(\nabla_\mu T)(\nabla_\nu T) + \frac{\mu}{T}\,\Sigma_\rho - \frac{1}{T}\,u_\nu Q^\nu \;\geq\; 0. \tag{2.49}$$

Positivity is secured if $\hat\eta, \zeta, \kappa_T \geq 0$ and $u_\nu Q^\nu \leq 0$, consistent with the spectral viscosity field $\eta > 0$ selecting the arrow of time.

2.11.6 Kubo formulas and transport from spectral correlators

Transport coefficients are obtained from retarded correlators of the microscopic stress and currents in the ordered background (Vol. I, Ch. 4):

$$\hat\eta \;=\; -\lim_{\omega\to 0}\frac{1}{\omega}\,\mathrm{Im}\,G^R_{T^{xy}T^{xy}}(\omega, \mathbf{k}=0), \qquad \zeta \;=\; -\lim_{\omega\to 0}\frac{1}{\omega}\,\mathrm{Im}\,G^R_{T^{i}{}_{i}\,T^{j}{}_{j}}(\omega, 0), \tag{2.50}$$

$$\kappa_T \;=\; -\lim_{\omega\to 0}\frac{1}{\omega T}\,\mathrm{Im}\,G^R_{J^Q_i J^Q_i}(\omega, 0), \qquad D_\rho \;=\; \lim_{\omega\to 0}\frac{-1}{\omega}\,\mathrm{Im}\,\frac{G^R_{J_iJ_i}(\omega,0)}{\chi_{JJ}}, \tag{2.51}$$

with J^Q the heat current and χ_{JJ} a static susceptibility. All coefficients become functions of the spectral state $\mathcal{S}$.

2.11.7 Compact continuity sheet (aether frame)

In the u^μ frame, the balance equations take the familiar divergence form:

$$\partial_t\varepsilon + \nabla\cdot(\varepsilon\,\mathbf{v} + \mathbf{q}) + \pi^{ij}\sigma_{ij} + p\,\nabla\cdot\mathbf{v} \;=\; -\,Q^0, \tag{2.52}$$

$$\partial_t(\rho) + \nabla\cdot(\rho\,\mathbf{v} + \boldsymbol{\nu}) \;=\; \Sigma_\rho, \tag{2.53}$$

$$\partial_t\mathbf{p} + \nabla\cdot\mathbf{\Pi} \;=\; \mathbf{Q}, \qquad \mathbf{\Pi} = p\,\mathbb{1} + \pi - \mathbf{v}\otimes\mathbf{q}, \tag{2.54}$$

$$\partial_t s + \nabla\cdot\left(s\,\mathbf{v} - \frac{\mathbf{q}}{T} - \frac{\mu}{T}\,\boldsymbol{\nu}\right) \;=\; \sigma_{\mathrm{diss}} \;\geq 0, \tag{2.55}$$

with $\mathbf{v}$ the three-velocity, $\mathbf{p}$ momentum density, and σ_{diss} given by Eq. (2.49). These reduce to the standard conservative/hydrodynamic forms when $Q^\nu = \Sigma_\rho = 0$ and transport is neglected.

2.11.8 Consistency Gates Crosswalk

We summarize how each balance law maps to the global consistency gates **S** (stability/hyperbolicity), **G** (gauge/Ward), **TQ** (topological/quantization), and **D** (dissipation/arrow), together with operational diagnostics for analytics and numerics.

Numerical gate checklist (to be run every step / output slice).

- **S (stability/hyperbolicity):** eigenvalues of K strictly positive; principal symbol cone inside that of $g_{\rm eff}$; $c_s^2, c_T^2 \in (0,1]$.
- **G (gauge/Ward):** residuals $\|\nabla_\mu J^\mu + \Gamma_{\rm leak}\|_\infty < \varepsilon_{\rm ward}$; for EM, monitor $\|\nabla_\mu T^{\mu\nu}_{\rm matter+EM} - Q^\nu\|_\infty$.
- **TQ (topological):** conserved winding/Hopf charges constant to tolerance under refinement; flux quantization on closed surfaces.
- **D (dissipation/arrow):** cell-wise $\sigma_{\rm diss} \geq 0$ from Eq. (2.49); cumulative entropy nondecreasing.

Restoration targets. In regimes with $\eta \downarrow 0$ and weak anisotropy, verify recovery of isotropic single-clock EFT: $c_\parallel \to c_\perp = c_s$, $c_T \to 1$, $\Gamma_{\rm leak} \to 0$, and standard adiabatic closures for (ε, p).

2.12 Hydrodynamics and Elasticity of the Spectral Medium

2.12.1 From the structural Lagrangian to balance laws

Starting from the unified structural Lagrangian of Vol. I, the coarse–grained, near–equilibrium dynamics is captured by a relativistic continuum on the emergent geometry $g^{\rm eff}_{\mu\nu}[\mathcal{S}]$ with flow $u^\mu u_\mu = -1$. Define the spatial projector $\Delta^{\mu\nu} \equiv g^{{\rm eff}\,\mu\nu} + u^\mu u^\nu$, expansion $\theta \equiv \nabla_\mu u^\mu$, shear $\sigma^{\mu\nu} \equiv \Delta^{\mu\alpha}\Delta^{\nu\beta}\big(\nabla_{(\alpha}u_{\beta)} - \frac{1}{3}\Delta_{\alpha\beta}\theta\big)$, and vorticity $\omega^{\mu\nu} \equiv \Delta^{\mu\alpha}\Delta^{\nu\beta}\nabla_{[\alpha}u_{\beta]}$. The balance laws read

$$\nabla_\mu T^{\mu\nu} = Q^\nu, \qquad \nabla_\mu N^\mu = \Sigma_\rho, \qquad \nabla_\mu J^\mu = -\,\Gamma_{\rm leak}, \tag{2.56}$$

where $T^{\mu\nu}$ is the total (matter+structural) stress–energy, N^μ the mass/number current, J^μ a conserved charge current in the closed limit, and Q^ν encodes open channels to sub-sectors (Sec. 1.4).

Constitutive split. Decompose

$$T^{\mu\nu} = \varepsilon\, u^\mu u^\nu + (p + \Pi)\,\Delta^{\mu\nu} + \pi^{\mu\nu} + u^\mu q^\nu + u^\nu q^\mu, \tag{2.57}$$

with energy density ε, equilibrium pressure p, bulk correction Π, heat flux $q^\mu u_\mu = 0$, and traceless viscous stress $\pi^\mu{}_\mu = 0$, $\pi^{\mu\nu}u_\nu = 0$. Anisotropy enters through the unit director $n^\mu \equiv \Delta^\mu{}_\nu \hat{\chi}^\nu / \sqrt{\hat{\chi}\cdot\hat{\chi}}$ with projectors $P^{\mu\nu}_\parallel \equiv n^\mu n^\nu$, $P^{\mu\nu}_\perp \equiv \Delta^{\mu\nu} - n^\mu n^\nu$.

2.12.2 Anisotropic transport and positivity

At first order in gradients (Navier–Stokes level), the most general *parity-even* constitutive relations compatible with the gates (Sec. 1.8) and with a single director are

$$\Pi = -\,\zeta\,\theta, \qquad \zeta \geq 0, \tag{2.58}$$

$$\pi^{\mu\nu} = -\,\eta_\perp\,\sigma^{\mu\nu} - (\eta_\parallel - \eta_\perp)\left(\sigma^{\alpha\beta}n_\alpha n_\beta\right)\left(P^{\mu\nu}_\parallel - \tfrac{1}{3}\Delta^{\mu\nu}\right), \tag{2.59}$$

$$q^\mu = -\,\kappa_\perp\,P^{\mu\nu}_\perp\left(\nabla_\nu T + T a_\nu\right) - \kappa_\parallel\,P^{\mu\nu}_\parallel\left(\nabla_\nu T + T a_\nu\right), \tag{2.60}$$

with four nonnegative transport coefficients $\zeta, \eta_\perp, \eta_\parallel, \kappa_{\perp,\parallel}$, acceleration $a^\mu \equiv u^\alpha \nabla_\alpha u^\mu$. In the spectral medium we parameterize

$$\eta_\perp = \alpha_\perp(\mathcal{S})\,\eta, \quad \eta_\parallel = \alpha_\parallel(\mathcal{S})\,\eta, \quad \zeta = \beta(\mathcal{S})\,\eta, \quad \kappa_{\perp,\parallel} = \gamma_{\perp,\parallel}(\mathcal{S})\,T,$$

so that dissipation is *monotone* in the viscosity field $\eta > 0$.

Entropy production. Define the entropy current $s^\mu = s\,u^\mu - \frac{q^\mu}{T}$. Then (to first order)

$$\nabla_\mu s^\mu \;=\; \frac{\zeta}{T}\,\theta^2 + \frac{\eta_\perp}{2T}\,\sigma_{\mu\nu}\sigma^{\mu\nu} + \frac{\eta_\parallel - \eta_\perp}{T}\left(\sigma_{\alpha\beta} n^\alpha n^\beta\right)^2 + \kappa_\perp \frac{(P_\perp^{\mu\nu}\nabla_\mu T)(P_{\perp\,\nu}{}^\lambda \nabla_\lambda T)}{T^2} + \kappa_\parallel \frac{(n^\mu \nabla_\mu T)^2}{T^2} \;\geq 0, \tag{2.61}$$

ensuring gate **D**.

2.12.3 Linear modes: sound and shear with anisotropy

Linearize Eqs. (2.56)–(2.60) around a homogeneous state $(\varepsilon_0, p_0, T_0, u^\mu = (1,\mathbf{0}), n^\mu = (0,\hat{\mathbf{n}}))$ and seek plane waves $e^{-i\omega t + i\mathbf{k}\cdot\mathbf{x}}$. Write $k_\parallel \equiv \mathbf{k}\cdot\hat{\mathbf{n}}$, $k_\perp^2 \equiv k^2 - k_\parallel^2$, enthalpy $\mathcal{H} \equiv \varepsilon_0 + p_0$, and adiabatic compressibility $c_s^2 \equiv (\partial p/\partial \varepsilon)_{s/n}$.

Longitudinal (sound) branch. To leading order in k,

$$\omega_{\mathrm{L}} \;=\; c_s(\theta)\,k \;-\; i\,\Gamma_{\mathrm{L}}(\theta)\,k^2 \;+\; \mathcal{O}(k^3), \qquad \cos\theta \equiv k_\parallel/k, \tag{2.62}$$

with a mild angular dependence $c_s(\theta)$ inherited from the structural sector (Vol. II) and attenuation

$$\Gamma_{\mathrm{L}}(\theta) \;=\; \frac{1}{2\mathcal{H}}\left[\zeta + \tfrac{4}{3}\eta_\perp + \tfrac{4}{3}(\eta_\parallel - \eta_\perp)\cos^2\theta\right]. \tag{2.63}$$

Transverse (shear) branch. Two degenerate shear polarizations in the isotropic limit split as

$$\omega_{\mathrm{T}} \;=\; -\,i\,\nu_\perp\,k_\perp^2 - i\,\nu_\parallel\,k_\parallel^2 + \mathcal{O}(k^3), \qquad \nu_{\perp,\parallel} \equiv \frac{\eta_{\perp,\parallel}}{\mathcal{H}}, \tag{2.64}$$

with purely diffusive decay at small k. Causality (gate **S**) requires these diffusions to be promoted to second–order (Israel–Stewart) form at high frequency to avoid superluminal fronts; we include relaxation times $\tau_{\pi,\perp/\parallel}$ so that $\tau_\pi \dot{\pi}^{\mu\nu} + \pi^{\mu\nu} = \cdots$.

2.12.4 Elastic limit and spectral moduli

When the structural sector freezes partially (solid-like phase), introduce a displacement field ξ_i comoving with u^μ and small strain $u_{ij} = \frac{1}{2}(\nabla_i \xi_j + \nabla_j \xi_i)$. Anisotropic Hooke law with Lamé moduli $\lambda_{\perp,\parallel}$, shear moduli $\mu_{\perp,\parallel}$,

$$\sigma_{ij} \;=\; \lambda_\perp\,u^k{}_k\,\delta_{ij} + (\lambda_\parallel - \lambda_\perp)\,u_{\ell m} n^\ell n^m\,\delta_{ij} + 2\mu_\perp\,u_{ij} + 2(\mu_\parallel - \mu_\perp)\,u_{k(i}n_{j)}n^k, \tag{2.65}$$

yields wave speeds (in the rest frame)

$$c_T^2(\theta) = \frac{\mu_\perp + (\mu_\parallel - \mu_\perp)\cos^2\theta}{\mathcal{H}}, \tag{2.66}$$

$$c_L^2(\theta) = \frac{K_\perp + (K_\parallel - K_\perp)\cos^2\theta + \frac{4}{3}\,\mu_\perp + \frac{4}{3}(\mu_\parallel - \mu_\perp)\cos^2\theta}{\mathcal{H}}, \qquad K \equiv \lambda + \tfrac{2}{3}\mu, \tag{2.67}$$

which smoothly match the hydrodynamic $c_s(\theta)$ in the fluid limit. Spectral moduli (λ, μ) are functionals of the quadratic curvature/stiffness matrices of the structural Lagrangian (Vol. I, Ch. 3).

2.12.5 Shock discontinuities and Rankine–Hugoniot conditions

Across a noncharacteristic hypersurface Σ with normal n_μ, the jump conditions are

$$\Big[N^\mu n_\mu\Big]_\Sigma = 0, \qquad \Big[T^{\mu\nu} n_\nu\Big]_\Sigma = 0, \tag{2.68}$$

supplemented by the entropy inequality $\Big[s\, u^\mu n_\mu\Big]_\Sigma \geq 0$. Anisotropy enters via the orientation of n^μ relative to n^μ (director): oblique shocks acquire different viscous/thermal substructures governed by $\eta_{\perp,\parallel}, \kappa_{\perp,\parallel}$; admissibility requires $\sigma_{\rm diss} \geq 0$ in the internal layer.

2.12.6 Conductivity and Onsager matrix

For a conserved charge with chemical potential μ, define the conduction current (Landau frame):

$$J^\mu = n_q u^\mu + \Sigma^{\mu\nu}\Big(E_\nu - T\nabla_\nu(\mu/T)\Big), \qquad \Sigma^{\mu\nu} = \sigma_\perp\, P_\perp^{\mu\nu} + \sigma_\parallel\, P_\parallel^{\mu\nu}, \tag{2.69}$$

with $E_\mu \equiv F_{\mu\nu}u^\nu$. The 2×2 transport block $\{\sigma_{\perp,\parallel}, \kappa_{\perp,\parallel}\}$ and thermo-electric cross terms obey Onsager reciprocity and positivity of the quadratic dissipation functional, thereby ensuring gate **D**.

2.12.7 Gate summary and spectral closure

- **S (stability/causality):** promote first-order theory by causal relaxation times $\tau_\pi, \tau_q > 0$; require $0 < c_{L,T}^2(\theta) \leq 1$ and front velocities within the cone of $g_{\rm eff}$.
- **G (gauge/Ward):** if coupled to gauge sectors, $\nabla_\mu(T^{\mu\nu}_{\rm matter} + T^{\mu\nu}_{\rm gauge}) = 0$ in the closed limit.
- **TQ (topological):** in solid-like phases, Burgers/defect densities provide conserved charges whose fluxes match jump conditions (2.68).
- **D (dissipation/arrow):** nonnegative $\zeta, \eta_{\perp,\parallel}, \kappa_{\perp,\parallel}, \sigma_{\perp,\parallel}$ guarantee $\nabla_\mu s^\mu \geq 0$ (Eq. (2.61)).

Spectral linkage. All transport coefficients are *functions of* the spectral fields $(\rho, \eta, \Phi, \chi^\mu, T)$; to leading order it suffices to treat them as $\mathcal{C}^1$ functions of η and ρ with mild anisotropy factors along n^μ. This closes the hydrodynamic/elastic description on the emergent geometry $g_{\rm eff}[\mathcal{S}]$ and provides initial data and priors for the optical (Sec. 2.13), spin/MPD (Sec. **??**), and GKSL radiation–reaction (Sec. **??**) sections.

2.13 Geometric and Wave Optics on the Emergent Metric

2.13.1 Eikonal limit and Fermat's principle on $g_{\rm eff}$

Consider a (nearly) massless spectral excitation φ (scalar, or a transverse mode of a gauge/emergent-phonon sector) governed at quadratic order by

$$\mathcal{L}_\varphi^{(2)} \;=\; \tfrac{1}{2}\,K(\mathcal{S})\,(\partial_t\varphi)^2 - \tfrac{1}{2}\,Z^{ij}(\mathcal{S};\hat\chi)\,\nabla_i\varphi\,\nabla_j\varphi - \tfrac{1}{2}\,m_{\rm eff}^2(\mathcal{S})\,\varphi^2 \qquad (m_{\rm eff}\approx 0 \text{ for optics}). \tag{2.70}$$

In the WKB/eikonal regime set $\varphi(x) = A(x)e^{i\Theta(x)/\epsilon}$, $\epsilon \downarrow 0$, with wave covector $k_\mu \equiv \nabla_\mu \Theta$. At leading order the *Hamilton–Jacobi* (dispersion) relation reads

$$\mathcal{H}(x,k) \;\equiv\; g^{\mu\nu}_{\text{eff}}(x)\, k_\mu k_\nu \;+\; \Pi^{\mu\nu}(x;\hat{\chi})\, k_\mu k_\nu \;=\; 0, \qquad \Pi^{0\mu} = 0, \quad \Pi^{ij} \equiv \frac{Z^{ij}}{K} - g^{ij}_{\text{eff}}, \tag{2.71}$$

which defines an *anisotropic indicatrix* in momentum space. Rays are the bicharacteristics of $\mathcal{H}$:

$$\frac{dx^\mu}{d\lambda} \;=\; \frac{\partial \mathcal{H}}{\partial k_\mu}, \qquad \frac{dk_\mu}{d\lambda} \;=\; -\frac{\partial \mathcal{H}}{\partial x^\mu}, \tag{2.72}$$

reducing to null geodesics of g_{eff} when $\Pi^{\mu\nu} = 0$. Equivalently, a Fermat principle holds: $\delta \int n(x, \hat{\mathbf{k}})\, ds = 0$ with direction-dependent *refractive index* n defined below.

2.13.2 Effective refractive index and birefringence

Let u^μ be the aether frame, split $k^\mu = \omega u^\mu + k^\mu_\perp$, and decompose with the director n^μ: $k_\parallel \equiv k^\mu_\perp n_\mu$, $k^2_\perp \equiv k^\mu_\perp k^\perp{}_\mu - k^2_\parallel$. For a two-polarization sector with principal symbol $Z^{ij} = \text{diag}(c^2_\perp, c^2_\perp, c^2_\parallel)$ in a local orthonormal triad, the dispersion is

$$\omega^2 \;=\; c^2_\parallel\, k^2_\parallel + c^2_\perp\, k^2_\perp, \qquad 0 < c^2_\perp \le c^2_\parallel \le 1, \tag{2.73}$$

yielding phase and group indices

$$n_{\text{ph}}(\theta) \equiv \frac{1}{c_{\text{ph}}(\theta)} = \frac{k}{\omega} = \Big(c^2_\parallel \cos^2\theta + c^2_\perp \sin^2\theta\Big)^{-1/2}, \quad \mathbf{v}_{\text{g}} = \nabla_{\mathbf{k}}\omega, \tag{2.74}$$

with $\cos\theta \equiv k_\parallel/k$. If the two polarizations experience distinct $(c_\perp, c_\parallel)$ one obtains *birefringence* with two indicatrices $n^{(A)}(\theta)$, $n^{(B)}(\theta)$ and polarization-dependent rays.

2.13.3 Transport of amplitude and polarization

Next order in the WKB expansion gives *Liouville transport* along rays:

$$\frac{d}{d\lambda}\Big(A^2\, \mathcal{J}\Big) = -\,2\,\Gamma_{\text{opt}}\, A^2\, \mathcal{J}, \qquad \mathcal{J} \equiv \Big|\det(\partial x_\perp/\partial k_\perp)\Big|, \tag{2.75}$$

where $\mathcal{J}^{-1}$ is the bundle expansion of the ray congruence and $\Gamma_{\text{opt}} \ge 0$ encodes weak absorption from $\eta > 0$. Polarization is parallel-transported by an effective Berry connection $\mathcal{A}_\mu(\hat{\mathbf{k}})$; in slowly varying media the Pancharatnam–Berry phase is $\Delta\phi = \oint \mathcal{A}_\mu\, dx^\mu$ and produces small rotation of the polarization ellipse.

2.13.4 Lensing and ray congruences

Let k^μ be a null-like tangent (for optics) normalized by $k^\mu u_\mu = -\omega$. Decompose the screen-projected deformation of a congruence into expansion Θ, shear Σ_{ab}, and twist Ω_{ab}. The Sachs equations generalize to

$$\frac{d\Theta}{d\lambda} = -\,\Theta^2 - \text{Tr}\,\Sigma^2 + \text{Tr}\,\Omega^2 - \tfrac{1}{2}\,\mathcal{R}_{\text{opt}}, \tag{2.76}$$

$$\frac{d\Sigma_{ab}}{d\lambda} = -2\Theta\,\Sigma_{ab} - C^{\text{opt}}_{acbd}\, e^c e^d, \qquad \mathcal{R}_{\text{opt}} \equiv R^{\text{eff}}_{\mu\nu} k^\mu k^\nu + \Delta_\Pi, \tag{2.77}$$

where $R^{\text{eff}}_{\mu\nu}$ is the Ricci of g_{eff}, C^{opt} the Weyl-like tidal tensor on the screen, and Δ_Π the anisotropic focusing term induced by $\Pi^{\mu\nu}$ in (2.71). Weak-lensing observables (convergence, shear, rotation) then follow from line-of-sight integrals of these optical scalars, supplying a direct mapping from spectral couplings to CMB/LSS lensing kernels (cf. App. **??**).

2.13.5 Interfaces: anisotropic Snell–Descartes laws

Across an interface Σ where $(c_\perp, c_\parallel, n^\mu)$ jump but g_{eff} is continuous, phase matching imposes continuity of tangential wavevector: $[k_t]_\Sigma = 0$, while the normal component obeys the Fresnel equation derived from (2.73). For an incidence plane containing n^i, generalized Snell laws read

$$\frac{\sin\theta_t}{n^{(p)}_{\text{ph}}(\theta_t)} = \frac{\sin\theta_i}{n^{(p)}_{\text{ph}}(\theta_i)}, \qquad p \in \{A, B\}, \tag{2.78}$$

with polarization-dependent transmission angles θ_t. Fresnel coefficients follow from continuity of $(\varphi, \partial_n\varphi)$ (or the appropriate electromagnetic-like pairs), featuring ordinary/extraordinary branches in the birefringent case.

2.13.6 Caustics, envelopes, and validity

Eikonal theory fails at caustics where $\mathcal{J} \to 0$. Uniform asymptotics (Airy/Pearcey) in local canonical coordinates heal the divergence and predict intensity scalings $I \sim \epsilon^{-2/3}$ (fold), $I \sim \epsilon^{-1/2}$ (cusp). The anisotropy modifies the catastrophe set via the indicatrix curvature $\kappa(\theta) = \partial^2_\theta n^{-1}_{\text{ph}}(\theta)$.

2.13.7 Gate summary for optics

- **S (stability/causality).** Require $0 < c_\perp^2 \le c_\parallel^2 \le 1$ and front velocity ≤ 1 in the g_{eff} cone; $\Gamma_{\text{opt}} \ge 0$.
- **G (gauge/Ward).** For electromagnetic-like sectors, impose the transversality/constraint structure consistent with emergent gauge isometries.
- **TQ (topological).** Ray bundles encircling structural defects acquire quantized Berry phases; caustic classifications (fold/cusp/swallowtail) provide robust, topology-protected signatures.
- **D (dissipation/arrow).** Weak absorption from $\eta > 0$ ensures monotone entropy production along propagation paths and regularizes caustic divergences at finite resolution.

Spectral linkage. The optical data $\{c_\perp, c_\parallel, \Gamma_{\text{opt}}\}$ and the index indicatrix $n_{\text{ph}}(\theta)$ are functionals of the spectral state $\mathcal{S} = \{\Phi, \rho, \eta, \chi^\mu, T\}$ via Z^{ij}/K in (2.70). In the isotropic restoration limit ($\eta \downarrow 0$, $\hat{\chi}$ randomized) one recovers null geodesics of g_{eff} and standard lensing kernels.

2.14 Rigid-Body Rotation and Classical Spin

2.14.1 Extended bodies on g_{eff} and MPD-type dynamics

Consider a compact extended configuration of the spectral medium whose rest-frame mass density is dominated by ρ and whose internal phase/current structure Φ, χ^μ yields a finite intrinsic angular momentum density. Let $x^\mu(\tau)$ denote a representative worldline (center-of-mass in the Tulczyjew–Dixon sense) with $u^\mu \equiv dx^\mu/d\tau$ normalized by $g_{\text{eff}}(u,u) = -1$. Define the effective momentum p^μ and spin tensor $S^{\mu\nu} = -S^{\nu\mu}$ by the covariant multipole

expansion of the (coarse-grained) stress-energy $T^{\mu\nu}$ on $g_{\rm eff}$. To pole-dipole order one obtains generalized Mathisson–Papapetrou–Dixon (MPD) equations on the emergent geometry (cf. GR case; here with spectral corrections):

$$\frac{Dp^\mu}{d\tau} = -\frac{1}{2}\, R^\mu{}_{\nu\alpha\beta}[g_{\rm eff}]\, u^\nu S^{\alpha\beta} \;+\; F^\mu_{(\chi)} \;+\; F^\mu_{(\eta)} \;+\; \mathcal{O}(\nabla\nabla T^{\rm int}), \tag{2.79}$$

$$\frac{DS^{\mu\nu}}{d\tau} = p^\mu u^\nu - p^\nu u^\mu \;+\; \tau^{\mu\nu}_{(\chi)} \;+\; \tau^{\mu\nu}_{(\eta)} \;+\; \mathcal{O}(\nabla T^{\rm int}), \tag{2.80}$$

supplemented by a spin supplementary condition (SSC), e.g.

$$\text{Tulczyjew–Dixon SSC:}\quad S^{\mu\nu}p_\nu = 0 \qquad \text{or} \qquad \text{Pirani SSC:}\quad S^{\mu\nu}u_\nu = 0. \tag{2.81}$$

Here $R^\mu{}_{\nu\alpha\beta}[g_{\rm eff}]$ is the Riemann tensor of the emergent metric (Vol. I, Ch. 2–3). The forces/torques $F^\mu_{(\chi)}, \tau^{\mu\nu}_{(\chi)}$ arise from couplings between spin and the internal director χ^μ; $F^\mu_{(\eta)}, \tau^{\mu\nu}_{(\eta)}$ encode dissipative spin-down from spectral viscosity $\eta > 0$. At leading order they follow from the effective interaction density

$$\mathcal{L}^{\rm spin}_{\rm int} = \frac{\alpha_\chi}{2}\, S^{\mu\nu}\, \nabla_\mu \chi_\nu + \frac{\beta_\chi}{2}\, S^{\mu\nu}\, \chi_\mu \chi_\nu \;-\; \frac{\gamma_\eta}{4}\, S^{\mu\nu} S_{\mu\nu}, \tag{2.82}$$

so that (schematically)

$$\tau^{\mu\nu}_{(\chi)} = 2\,\frac{\partial \mathcal{L}^{\rm spin}_{\rm int}}{\partial S_{\mu\nu}} = \alpha_\chi\, \nabla^{[\mu}\chi^{\nu]} + \beta_\chi\, \chi^{[\mu}\chi^{\nu]}, \qquad \tau^{\mu\nu}_{(\eta)} = -\,\gamma_\eta\, S^{\mu\nu}, \tag{2.83}$$

and $F^\mu_{(\chi)}$ follows from the covariant divergence of the spin-coupled stress. The $\gamma_\eta > 0$ term produces angular-momentum damping consistent with the entropy gate (Sec. 1.4.4).

Define the (spin) magnitude $S^2 \equiv \frac{1}{2} S^{\mu\nu} S_{\mu\nu}$; under TD-SSC and in absence of dissipation, S^2 is conserved at the pole-dipole level. The effective rest mass $m \equiv \sqrt{-p^\mu p_\mu}$ is constant up to work done by $F^\mu_{(\chi)} u_\mu$ and dissipative terms.

2.14.2 Rigid-body kinematics and Euler equations on $g_{\rm eff}$

In the aether rest frame u^μ, consider a bounded body occupying a domain $\mathcal{B}_t$ with mass density $\rho(\mathbf{x}, t)$ and velocity field $\mathbf{v}(\mathbf{x}, t)$, with small internal strain so that the motion is well approximated by a rigid rotation plus translation: $\mathbf{v}(\mathbf{x}) = \mathbf{V} + \boldsymbol{\Omega} \times (\mathbf{x} - \mathbf{X})$. Define the (screen-projected) inertia tensor on the spatial slice orthogonal to u^μ with metric $\gamma_{ij} = g^{\rm eff}_{ij} + u_i u_j$,

$$I_{ij} = \int_{\mathcal{B}_t} d^3x\, \rho \left(\delta_{ij}\, r^2 - r_i r_j\right), \qquad \mathbf{r} \equiv \mathbf{x} - \mathbf{X}, \tag{2.84}$$

and angular momentum $L_i \equiv I_{ij}\Omega^j$. On a slowly varying background congruence (expansion θ, shear σ_{ij}, vorticity ω_{ij} of u^μ), the covariant Euler equations generalize to

$$\frac{DL_i}{dt} + \epsilon_{ijk}\, \Omega^j L^k + \underbrace{\left(\sigma_{ij} + \tfrac{1}{3}\theta\, \delta_{ij}\right) L^j}_{\text{cosmogeometric drag}} = \tau^{\rm ext}_i - \underbrace{\nu_{\rm rot}\, L_i}_{\text{viscous spin-down}}, \tag{2.85}$$

where D/dt is the convective derivative along the body worldline, $\tau^{\rm ext}_i$ collects external torques (including tidal terms from $R^\mu{}_{\nu\alpha\beta}$ projected on the body), and $\nu_{\rm rot} \sim \eta/\rho R^2$ is an effective rotational damping rate set by the spectral viscosity and body size R. Equation (2.85) reduces to the classical Euler equations in flat, static backgrounds ($\theta = \sigma_{ij} = 0$) and $\nu_{\rm rot} = 0$.

2.14.3 Gyroscope precession and spin–curvature/director couplings

Introduce an orthonormal tetrad $e_a{}^\mu$ comoving with the body, $e_0{}^\mu = u^\mu$, and define the spin 4-vector $S^\mu \equiv \frac{1}{2}\,\epsilon^{\mu\nu\alpha\beta} u_\nu S_{\alpha\beta}$. Projecting (2.80) one finds the precession law

$$\frac{DS^\mu}{d\tau} = \Omega^\mu{}_\nu\, S^\nu - \gamma_\eta\, S^\mu + \mathcal{O}(S\,\nabla\nabla\chi), \tag{2.86}$$

with the precession tensor

$$\Omega_{\mu\nu} = \underbrace{\omega^{\mathrm{FW}}_{\mu\nu}}_{\text{Fermi–Walker}} + \underbrace{\Omega^{\mathrm{curv}}_{\mu\nu}}_{\propto R_{\mu\nu\alpha\beta}u^\alpha u^\beta} + \underbrace{\Omega^{(\chi)}_{\mu\nu}}_{\propto \alpha_\chi \nabla_{[\mu}\chi_{\nu]} + \beta_\chi \chi_{[\mu}\chi_{\nu]}} . \tag{2.87}$$

Here $\omega^{\mathrm{FW}}_{\mu\nu}$ encodes Thomas/de Sitter-like effects on g_{eff}, while $\Omega^{(\chi)}_{\mu\nu}$ captures precession induced by spatial anisotropy and gradients of the internal director. In the weak-field, slow-motion limit (aether frame) one obtains

$$\dot{\mathbf{S}} = \boldsymbol{\Omega}_{\mathrm{grav}} \times \mathbf{S} + \boldsymbol{\Omega}_\chi \times \mathbf{S} - \gamma_\eta\, \mathbf{S}, \qquad \boldsymbol{\Omega}_\chi \simeq \tfrac{1}{2}\,\alpha_\chi\, \nabla \times \boldsymbol{\chi} + \beta_\chi\, (\boldsymbol{\chi}\cdot\nabla)\,\boldsymbol{\chi}, \tag{2.88}$$

with $\boldsymbol{\Omega}_{\mathrm{grav}}$ the usual (effective) geodetic + frame-drag frequency computed from g_{eff}. Equation (2.88) predicts small, environment-dependent deviations from GR gyroscope precession when χ^μ has coherent gradients.

2.14.4 Classical spin from internal circulation

A *classical spin* can be generated from quantized or coherent circulation of the spectral phase Φ within the body. For an axisymmetric configuration with phase winding $n \in \mathbb{Z}$ around $\hat{\mathbf{z}}$ and core scale ℓ, the intrinsic angular momentum scales as

$$S_{\mathrm{int}} \;\simeq\; \mathcal{I}_\Phi\, n, \qquad \mathcal{I}_\Phi \sim \int d^3x\, \rho\, \ell^2, \tag{2.89}$$

modulated by the stiffness coefficients of the Φ-sector (Vol. I, Ch. 4). Couplings α_χ, β_χ distinguish the alignment of S_{int} relative to χ^μ, selecting easy-axis/easy-plane behaviors for spinning solitons, with metastable states when χ^μ is frustrated by boundaries.

2.14.5 Stability, bounds, and gates

- **S (no ghosts, causal transport).** The kinetic blocks defining p^μ and the quadratic form of rotational perturbations around the body must be positive; front velocities of torsional waves ≤ 1 (in the g_{eff} cone).
- **G (gauge/Ward).** Spin couplings respect emergent isometries: $\nabla_\mu T^{\mu\nu} = 0$ on g_{eff}, and the director coupling $\mathcal{L}^{\mathrm{spin}}_{\mathrm{int}}$ must be built from covariant scalars under fiber isometries.
- **TQ (topological).** Phase windings n and Hopf charges of spinning textures protect S_{int} against dissipation up to barrier crossing; interfaces can pin or unpin n.
- **D (dissipation/arrow).** $\gamma_\eta \geq 0$ ensures $d(S^2)/d\tau \leq 0$ for isolated bodies; in steady torqued states, input power balances η-controlled losses.

Observational handles. Equation (2.88) implies environment-dependent gyroscope precession and spin-down in media with structured χ^μ or enhanced η. Precision tests include: (i) ring-laser gyros and atom interferometers in anisotropic metamaterials that emulate nonzero $\nabla \times \boldsymbol{\chi}$; (ii) spin evolution of compact, soliton-like objects in numerical simulations of the spectral medium; (iii) astrophysical precession of spinning dark solitons (Vol. II, Ch. **??**) in filamentary environments (cosmic web).

2.15 Radiation Reaction and Consistent Damping (GKSL)

2.15.1 Radiative channels of the spectral medium

An accelerated, compact excitation embedded in the spectral medium $\mathcal{S} = \{\Phi, \rho, \eta, \chi^\mu, T\}$ emits energy into propagating modes (phase/structural waves and tensorial deformations of $g_{\rm eff}$). Denote collectively by φ_A the radiative fields (A indexes polarizations/sectors) with quadratic action

$$S^{(2)}_{\rm rad}[\varphi_A] = \frac{1}{2}\int d^4x\sqrt{-g_{\rm eff}}\;\varphi_A\,\hat{\mathcal{D}}^{AB}\,\varphi_B, \qquad \hat{\mathcal{D}}^{AB} = \mathsf{K}^{AB}\nabla_t^2 + \mathsf{Z}^{AB\,ij}\nabla_i\nabla_j + \cdots, \tag{2.90}$$

and linear coupling to the source worldline $x^\mu(\tau)$,

$$S^{(\rm rad)}_{\rm int} = \int d\tau\; J_A(\tau)\,\varphi_A(x(\tau)). \tag{2.91}$$

Integrating out φ_A at tree level with the *retarded* Green function $G^{AB}_{\rm ret}$ produces a self-action

$$S_{\rm self} = \frac{1}{2}\int d\tau\, d\tau'\; J_A(\tau)\,G^{AB}_{\rm ret}\Big(x(\tau), x(\tau')\Big)\,J_B(\tau'), \tag{2.92}$$

whose variation yields a nonlocal, causal self-force $F^\mu_{\rm rad}$.

2.15.2 Causal memory kernel and local reduction

Write the generalized equation of motion on $g_{\rm eff}$ (including external force $F^\mu_{\rm ext}$ and viscous term from η)

$$m\,a^\mu = F^\mu_{\rm ext} + \underbrace{\int_{-\infty}^{\tau^-} d\tau'\;\mathsf{K}^\mu{}_\nu(\tau-\tau')\,a^\nu(\tau')}_{F^\mu_{\rm rad}} - \nu_\eta\,\Delta^\mu{}_\nu\,u^\nu + \mathcal{O}(\text{tidal, spin}), \qquad \Delta^\mu{}_\nu \equiv \delta^\mu{}_\nu + u^\mu u_\nu, \tag{2.93}$$

where $u^\mu = dx^\mu/d\tau$, $a^\mu = Du^\mu/d\tau$, and the memory kernel $\mathsf{K}^\mu{}_\nu(\Delta\tau)$ is supported inside the $g_{\rm eff}$ light cone. For slowly varying motion relative to the kernel correlation time τ_c one obtains a *local* expansion

$$F^\mu_{\rm rad} \simeq m\,\tau_0\Big(\dot{a}^\mu + \alpha\,a^2 u^\mu + \beta\,\mathcal{P}^\mu{}_\nu\,R^\nu{}_\lambda u^\lambda\Big) + \mathcal{O}(\tau_c^2\,\partial^3), \qquad \mathcal{P}^\mu{}_\nu \equiv \Delta^\mu{}_\nu, \tag{2.94}$$

which generalizes the Abraham–Lorentz–Dirac (ALD) force to the emergent geometry. The time scale $\tau_0 > 0$ and coefficients (α, β) are determined by spectral susceptibilities and can be anisotropic via $\hat{\chi} \equiv \chi^\mu/\sqrt{\chi^2}$ (see below). Equation (2.94) is *causal* because

it originates from a retarded kernel; runaway/pre-acceleration pathologies are avoided by (i) keeping the integro-differential form (2.93) or (ii) using *reduction of order* (à la Landau–Lifshitz):

$$\dot{a}^\mu \;\to\; \frac{D}{d\tau}\Big(\frac{F^\mu_{\rm ext}}{m}\Big) + \mathcal{O}(\tau_0), \quad \text{inside } F^\mu_{\rm rad}. \tag{2.95}$$

Anisotropy. If $\mathsf{Z}^{AB\,ij}$ in (2.90) is anisotropic (Vol. II, Sec. **??**), the kernel decomposes as $\mathsf{K}^\mu{}_\nu = \tau_0^{\parallel}\,\Pi^\mu{}_\nu{}^{(\parallel)} + \tau_0^{\perp}\,\Pi^\mu{}_\nu{}^{(\perp)} + \cdots$ with projectors $\Pi^{(\parallel/\perp)}$ along/orthogonal to $\hat{\chi}$, leading to direction-dependent damping and Larmor power (below).

2.15.3 Power balance and Larmor-type formulae

Contract (2.93) with u_μ; using $u_\mu a^\mu = 0$ and $u_\mu \dot{a}^\mu = -a^2$, one finds the energy balance in the aether frame:

$$\frac{dE}{d\tau} = u_\mu F^\mu_{\rm ext} - \underbrace{P_{\rm rad}}_{\geq 0} - \underbrace{P_\eta}_{\geq 0}, \qquad P_\eta = \nu_\eta\, u_\mu \Delta^\mu{}_\nu u^\nu = \nu_\eta, \tag{2.96}$$

with radiated power

$$P_{\rm rad} = -\,u_\mu F^\mu_{\rm rad} \simeq m\,\tau_0\left(a^2 + \alpha\, a^2\, u^2\right) = m\,\tau_0\, a^2 \quad \text{(to leading order)}, \tag{2.97}$$

generalizing the Larmor formula. In the anisotropic case $P_{\rm rad} = m\left(\tau_0^{\parallel}\, a_{\parallel}^2 + \tau_0^{\perp}\, a_{\perp}^2\right)$ with $a_{\parallel/\perp} \equiv \Pi^{(\parallel/\perp)}_{\mu\nu} a^\mu a^\nu$.

2.15.4 GKSL limit and fluctuation–dissipation

At the open-quantum level (Sec. 1.4), the same retarded kernel produces a completely positive (GKSL) generator with jump operators L_a proportional to the source density that couples to φ_A. The Kubo–Martin–Schwinger (KMS) relation for the spectral medium yields a fluctuation–dissipation identity linking the anti-Hermitian part of the self-energy to $P_{\rm rad}$. In frequency space (linear response),

$$\tilde{F}^\mu_{\rm rad}(\omega) = \tilde{\Gamma}^\mu{}_\nu(\omega)\,\tilde{v}^\nu(\omega), \qquad \tilde{\Gamma}(\omega) = \underbrace{\mathrm{Re}\,\tilde{\Gamma}}_{\text{reactive}} + i\,\underbrace{\mathrm{Im}\,\tilde{\Gamma}}_{\text{dissipative}}, \tag{2.98}$$

with Kramers–Kronig relations ensuring causality. The GKSL rates $\gamma_a(\mathcal{S})$ are proportional to $\mathrm{Im}\,\tilde{\Gamma}(\omega)$ evaluated at the relevant Bohr frequencies; positivity of $\gamma_a \Leftrightarrow P_{\rm rad} \geq 0$.

2.15.5 Runaway avoidance and well-posedness

(i) Integro-differential formulation. Equation (2.93) with a kernel $\mathsf{K}(\Delta\tau)$ that decays on τ_c and is strictly retarded has a unique solution given past data on an interval of length τ_c.

(ii) Reduction of order (ROO). Using (2.95) converts third-derivative terms into $\mathcal{O}(\tau_0)$ corrections in forces; no pre-acceleration occurs if $\tau_0 \ll$ external timescale.

(iii) Consistency gates.

- **Causality:** kernel support inside the g_{eff} cone.
- **Positivity:** $\tau_0^{\parallel,\perp} \geq 0 \;\Rightarrow\; P_{\text{rad}} \geq 0$.
- **Sub-luminality:** reactive corrections do not tilt the principal symbol outside the g_{eff} cone (checked via front-velocity bounds).
- **No strong coupling:** $\tau_0 \omega \ll 1$ in the validity range (ω characteristic frequency), else EFT completion required.

2.15.6 Illustrative closure: Landau–Lifshitz form on g_{eff}

Implement ROO by replacing $\dot{a}^\mu$ in (2.94) with $D(F^\mu_{\text{ext}}/m)/d\tau$ and projecting orthogonally to u^μ:

$$m\, a^\mu = F^\mu_{\text{ext}} + m\, \tau_0\, \Delta^\mu{}_\nu \, \frac{D}{d\tau}\left(\frac{F^\nu_{\text{ext}}}{m}\right) + \mathcal{O}(\tau_0^2)\,, \tag{2.99}$$

plus anisotropic corrections $\propto \tau_0^{\parallel} - \tau_0^{\perp}$ if $\hat{\chi}$ is coherent. Equation (2.99) is manifestly second order and causal, matches the GKSL power loss (2.97), and is the recommended evolution law for numerical protocols (Part V).

Summary. Radiation reaction in the spectral medium is governed by a causal memory kernel derived from retarded Green functions of the radiative sectors. Its local limit recovers an ALD-type force on g_{eff} with anisotropic coefficients fixed by spectral susceptibilities. Energy balance yields a Larmor-type power with $P_{\text{rad}} \geq 0$, consistent with GKSL positivity. Runaway and pre-acceleration are absent when using the full nonlocal equation or its Landau–Lifshitz reduction, provided the consistency gates (causality, positivity, sub-luminality, no strong coupling) are satisfied.

2.16 Hamilton–Jacobi Limit with Validity Conditions

Setup. Let the quadratic sector around an ordered background S_0 read

$$\mathcal{L}^{(2)} = \tfrac{1}{2}\,\delta\mathbf{S}^\top\Big[\mathsf{K}\,\partial_t^2 + \mathsf{C}\,\partial_t + \mathsf{Z}^{ij}\nabla_i\nabla_j + \mathsf{M}^2\Big]\delta\mathbf{S},$$

with $\mathsf{K} \succ 0$ and principal symbol defining anisotropic cones (Vol. II, Sec. **??**). Consider a single canonically-normalized scalar mode φ (a normal-mode component of $\delta\mathbf{S}$) governed effectively by

$$\mathcal{D}\varphi \equiv \partial_t^2\varphi + \Gamma\,\partial_t\varphi - \nabla_i(c^{ij}\nabla_j\varphi) + m_{\text{eff}}^2\,\varphi = 0, \qquad c^{ij} = u^{02}\,(\mathsf{K}^{-1}\mathsf{Z}^{ij})_{\text{eff}}. \tag{2.100}$$

Eikonal/WKB ansatz. Let $\varepsilon \equiv \lambda_{\text{dB}}/L \ll 1$ with de Broglie wavelength λ_{dB} and background variation scale L. Seek

$$\varphi(x) = A(x)\,\exp\left[\frac{i}{\varepsilon}S(x)\right], \qquad A, S \in \mathbb{R}.$$

Collecting orders of ε yields:

(i) $\mathcal{O}(\varepsilon^{-2})$ (eikonal):

$$g^{\mu\nu}_{\text{eff}}\,\partial_\mu S\,\partial_\nu S \;=\; m^2_{\text{eff}}, \tag{2.101}$$

where $g^{\mu\nu}_{\text{eff}}$ is defined by the inverse principal symbol (isotropized average of c^{ij} in the u^μ-frame).
(ii) $\mathcal{O}(\varepsilon^{-1})$ (transport):

$$\nabla_\mu\Big(A^2\,v^\mu\Big) \;=\; -\,\Gamma\,A^2, \qquad v^\mu \equiv \frac{g^{\mu\nu}_{\text{eff}}\partial_\nu S}{m_{\text{eff}}}, \tag{2.102}$$

so that v^μ is the ray 4-velocity and $\Gamma \geq 0$ damps flux.

Hamilton–Jacobi theorem (uniform form). Assume:

(A1) Scale separation: $\varepsilon \equiv \lambda_{\text{dB}}/L \ll 1$ and $|\nabla g_{\text{eff}}|\,L \lesssim \mathcal{O}(1)$.

(A2) Weak anisotropy in the inflation/propagation epoch: $\Delta c \equiv \|c^{ij} - c_{\text{iso}}\delta^{ij}\|/c_{\text{iso}} \leq \delta_{\text{an}} \ll 1$.

(A3) Dissipation gate: $0 \leq \Gamma \ll m_{\text{eff}}$ and $\sigma_{\text{diss}} \geq 0$ (Vol. I).

(A4) Hyperbolicity/causality: front velocity within the cone of g_{eff}.

Then the exact solution of (2.100) admits the representation

$$\varphi(x) = A(x)\,e^{\frac{i}{\varepsilon}S(x)} + \mathcal{R}_\varepsilon(x),$$

with S solving (2.101) and A solving (2.102), and the remainder bounded as

$$\|\mathcal{R}_\varepsilon\|_{L^2(\Sigma_t)} \;\leq\; C\,\varepsilon\,e^{\,c_1 t/L}\,, \tag{2.103}$$

for $t \lesssim c_2 L\,\log(1/\varepsilon)$, with constants $C, c_{1,2}$ controlled by $(\|\nabla g_{\text{eff}}\|, \|\nabla m_{\text{eff}}\|, \Gamma)$. Hence, geodesic/ray dynamics defined by (2.101) approximates the spectral evolution over macroscopic times with $\mathcal{O}(\varepsilon)$ phase error and $\mathcal{O}(\varepsilon)$ amplitude error.

Validity sheet (ready to cite).

HJ & geometric optics valid if: $\varepsilon \ll 1$, $\Delta c \ll 1$, $\Gamma/m_{\text{eff}} \ll 1$,
Curvature scale: $L_{\text{geo}} \gg \lambda_{\text{dB}}$, Time of validity: $t \lesssim \mathcal{O}\Big(L\log(1/\varepsilon)\Big)$.

2.17 Targeted Numerical Matching: Three Benchmarks

2.17.1 Free fall / perihelion (timelike rays).

Ray equations from $H(x,p) = \frac{1}{2}(g^{\mu\nu}_{\text{eff}}p_\mu p_\nu - m^2_{\text{eff}}) = 0$:

$$\dot{x}^\mu = \partial_{p_\mu}H, \qquad \dot{p}_\mu = -\partial_{x^\mu}H,$$

with $g_{\text{eff}} = g_{\text{GR}} + \delta g(\mathcal{S})$ and $m_{\text{eff}} = m_0(1 + \delta_m(\rho, \Phi))$. Observable: advance per orbit

$$\Delta\varpi \;=\; \Delta\varpi_{\text{GR}}\,\Big(1 + \delta_\varpi\Big), \qquad \Delta\varpi_{\text{GR}} = \frac{6\pi GM}{a(1-e^2)}.$$

Here δ_ϖ is derived from the PN-like expansion of δg.

2.17.2 Light bending / Shapiro (null rays).

Set $m_{\rm eff} = 0$ and use the null Hamiltonian $g^{\mu\nu}_{\rm eff} p_\mu p_\nu = 0$. Angles and delays:

$$\alpha = \frac{4GM}{b}\,(1+\delta_\alpha), \qquad \Delta t_{\rm Shapiro} = 2GM \ln \frac{4 r_E r_R}{b^2}\,(1+\delta_S),$$

with δ_α, δ_S functions of $(\delta g, \Delta c)$.

2.17.3 Shear waves & anti/normal birefringence.

For a transverse mode with $c_\parallel \neq c_\perp$:

$$\omega^2 = c_\parallel^2 k_\parallel^2 + c_\perp^2 k_\perp^2 + m_{\rm eff}^2, \qquad \Delta t(L,\theta) = L\left(\frac{1}{v_g(\theta)} - \frac{1}{\bar{v}_g}\right),$$

$v_g = \partial\omega/\partial k$. Compare Δt and polarization rotation with data; gate via $|\Delta c| \ll 1$.

Minimal integrator (pseudo-code).

```
Given g_eff(x), m_eff(x), initial (x0,p0):
function geodesic(T):
  x,p = x0,p0
  for n in 0..N-1:
    p += -dt * grad_x H(x,p)
    x +=  dt * grad_p H(x,p)
    p = project_on_mass_shell(p, g_eff, m_eff)  # constraint fix
  return trajectory
```

2.18 Parameter Map: Spectral → Classical

2.19 Strong Anisotropy: Birefringence Gates

Dispersion and cones. For two polarization families $A = \{\parallel, \perp\}$:

$$\omega_A^2 = c_A^2(\hat{\mathbf{k}})\, k^2 + m_A^2, \qquad |c_A - 1| \equiv \epsilon_A \ll 1 \text{ (gate)}.$$

Define $\Delta c = c_\parallel - c_\perp$. Statistical anisotropy in cosmological scalar modes scales as $g_* \sim \mathcal{O}(\Delta c)$; multi-messenger GW bounds imply $|\delta c_T| \ll 1$ at late times. We impose the *anisotropy gate*

$$|\Delta c| \leq \epsilon_{\rm aniso}, \qquad \epsilon_{\rm aniso} \text{ set by observational layer (CMB/LSS/GW).} \tag{2.104}$$

In the lab optics limit, polarization rotation after thickness L is

$$\Delta\phi(\theta) \simeq \tfrac{\omega L}{2}\,\frac{\Delta c(\theta)}{c_0},$$

which offers a direct metrology handle on $\Delta c(\theta)$ vs. $\hat{\chi}$.

2.20 Unified Wave Energy–Momentum with Causality/Positivity

Canonical fluxes. Let φ obey (2.100). The quadratic energy and Poynting-like flux in the u^μ-frame are

$$\mathcal{E} = \tfrac{1}{2}\Big[\dot\varphi^2 + (\nabla\varphi)^\top \mathsf{C}\,(\nabla\varphi) + m_{\rm eff}^2\varphi^2\Big], \tag{2.105}$$

$$\mathbf{P} = \dot\varphi\,\mathsf{C}\,\nabla\varphi, \qquad \mathsf{C} \equiv (c^{ij})\,. \tag{2.106}$$

Then

$$\partial_t\mathcal{E} + \nabla\cdot\mathbf{P} = -\,\Gamma\,\dot\varphi^{\,2} \;\le\; 0, \tag{2.107}$$

ensuring monotone decay (GKSL-consistent). Positivity gates: $\mathsf{C} \succ 0$ (no gradient instability), $m_{\rm eff}^2 \ge 0$ (no tachyon), and front velocities inside $g_{\rm eff}$ cone.

Radiation reaction for a compact soliton. For a charged soliton with 4-velocity U^μ and small damping ν,

$$m_{\rm eff}\,a^\mu = F^\mu{}_\nu U^\nu + \nu\left(\dot a^\mu + a^2 U^\mu\right) + \mathcal{O}(\nu^2), \tag{2.108}$$

the GKSL embedding (Sec. 1.4) guarantees no runaway/pre-acceleration within $\dot\nu/m_{\rm eff} \ll 1$ and energy balance with (2.107).

2.21 Guide to Classical Tests (Comparison Sheet)

Orbital tests.

$$\Delta\varpi = \frac{6\pi GM}{a(1-e^2)}\,(1+\delta_\varpi), \tag{2.109}$$

$$\Omega_{\rm LT}(r) = \frac{2GJ}{r^3}\,(1+\delta_{\rm LT}), \tag{2.110}$$

with $\delta_\varpi, \delta_{\rm LT}$ functions of $(M_{\rm eff}(\rho), \delta g, \Delta c)$.

Light propagation.

$$\alpha = \frac{4GM}{b}\,(1+\delta_\alpha), \tag{2.111}$$

$$\Delta t_{\rm Shapiro} = 2GM\ln\frac{4r_E r_R}{b^2}\,(1+\delta_S), \tag{2.112}$$

$$n(\omega,\theta) = \frac{1}{c_s(\theta)} = 1 + \delta_n(\omega,\theta). \tag{2.113}$$

Redshift & clocks.

$$1+z \;=\; \frac{(u_\mu p^\mu)_{\rm em}}{(u_\mu p^\mu)_{\rm rec}}\,(1+\delta_z), \qquad \delta_z \sim \partial_\rho M_{\rm eff}\,\delta\rho + \cdots\,.$$

Gravitational waves.

$$c_T = 1 + \delta c_T(\mathcal{S}), \qquad \dot{h} + \Big(3H + \nu_T\Big)h + c_T^2 k^2 h = 0, \tag{2.114}$$

$$r \simeq 16\,\epsilon_\eta\,\frac{c_s}{c_T}, \qquad \text{(Vol. II mapping to } \eta\text{-wave)}. \tag{2.115}$$

Checklist (pass/fail).

- **Gates:** $\mathsf{K} \succ 0$, $\mathsf{C} \succ 0$, $c_s^2, c_T^2 > 0$, front $\leq$ cone of g_{eff}.
- **Anisotropy:** $|\Delta c| \leq \epsilon_{\text{aniso}}$ from lab/CMB/GW layers.
- **Radiation:** energy balance (2.107); no runaways under (2.108).
- **Classical limits:** $\delta_\varpi, \delta_\alpha, \delta_S, \delta_{\text{LT}}, \delta c_T, \delta_n \to 0$ recover GR/SR.

Table 2.2: Representative high-precision classical constraints used as priors/gates. Bounds are indicative (68–95% CL) and serve to constrain the spectral parameter space; see text for caveats and mapping details.

Observable	**Quantity**	**Current bound**	**Probe**
Solar-conjunction Shapiro delay	$\lvert\gamma - 1\rvert$	$\lesssim 2 \times 10^{-5}$	Cassini
Light deflection (radio/VLBI)	$(1+\gamma)/2$	rel. acc. $\sim 10^{-4}$	VLBI
Perihelion advance (Mercury/Mars)	$\dfrac{2-\beta+2\gamma}{3}$	$\lesssim 10^{-3}$	ephemerides
Lunar Laser Ranging	$\left\lvert\dfrac{\dot{G}_{\rm eff}}{G_{\rm eff}}\right\rvert$	$\lesssim 10^{-13}$ yr^{-1}	LLR
Preferred-frame tests	$\lvert\alpha_1\rvert$, $\lvert\alpha_2\rvert$	$\lesssim 10^{-5}$ (typical)	pulsars/ephemerides
Gravitational redshift & time dilation	metric consistency	$\sim 10^{-5}$ rel. acc.	clock tests
GW speed (multi-messenger)	$\lvert\delta c_T\rvert$	$\lesssim 10^{-15}$	GW170817/GRB
Frame dragging (Lense–Thirring)	$\Omega_{\rm LT}$	$\sim$ (5–10)% rel. acc.	LAGEOS/LARES

Table 2.3: Conservation laws ↔ Consistency gates (S/G/TQ/D) with mathematical conditions and practical diagnostics.

Conservation / Balance	**Gate(s)**	**Mathematical condition(s)**	**Diagnostics / Failure modes**
Energy balance $u_\nu \nabla_\mu T^{\mu\nu} = u_\nu Q^\nu$	S, D	Time–kinetic blocks $\mathsf{K} \succ 0$; transport $\hat{\eta}, \zeta, \kappa_T \geq 0$; $u_\nu Q^\nu \leq 0$.	Negative $\nabla_\mu s^\mu$; ghost modes; runaway heating.
Momentum balance $\Delta^\alpha{}_\nu \nabla_\mu T^{\mu\nu} = \Delta^\alpha{}_\nu Q^\nu$	S, Caus.	Hyperbolicity of principal symbol; $c_s^2 > 0$, $c_T^2 > 0$; front velocity $\leq c_{\text{eff}}$.	Superluminal fronts; gradient instabilities; ill-posed PDEs.
Charge continuity $\nabla_\mu J^\mu = -\Gamma_{\text{leak}}$	G, TQ	Ward identities; anomaly cancellation; quantized fluxes in topological sectors; $\Gamma_{\text{leak}} \to 0$ in closed limit.	Gauge non-closure; spurious anomalies; unphysical leakage.
Mass/number continuity $\nabla_\mu N^\mu = \Sigma_\rho$	S, D	Diffusion $D_\rho \geq 0$; bounded sources $\lvert\Sigma_\rho\rvert$ sub-UV.	Negative diffusion; stiff advection–diffusion; mass drift.
Entropy production $\nabla_\mu s^\mu = \sigma_{\text{diss}} \geq 0$	D	Eq. (2.49) nonnegative for all flows.	$\sigma_{\text{diss}} < 0$ (analytical or numerical).
Angular momentum $\nabla_\lambda J^{\lambda\mu\nu} = (x^\mu Q^\nu - x^\nu Q^\mu) + \cdots$	S, G	Symmetric (Belinfante) $T^{\mu\nu}$; finite Q^ν; spin current well-defined.	Torque leakage; nonsymmetric $T^{\mu\nu}$ in closed limit.
Matter+EM closure $\nabla_\mu(T^{\mu\nu}_{\text{matter}} + T^{\mu\nu}_{\text{EM}}) = Q^\nu$	G, Caus.	Maxwell/Ward closure; $E\cdot B$ anomalies canceled; $\lvert\delta c_T\rvert$ within bounds.	Energy non-closure; $\lvert c_T - 1\rvert$ too large; causality violation.

Classical quantity	**Spectral expression / provenance**
Effective mass m_{eff}	$m_0 \left[1 + \alpha_m(\rho, \Phi) + \beta_m(\chi^\mu \chi^\nu D_\mu D_\nu)\right]$
Planck scale M_{eff}	$M_0 \left[1 + \alpha_P(\rho) + \cdots\right]$ (Vol. I, Ch. 2–3)
Refractive index $n(\omega, \theta)$	$n = \frac{1}{c_s(\theta)}$, $c_s^2 = \frac{\partial p}{\partial \epsilon}$ from Z^{ij}
Birefringence Δc	$c_\parallel - c_\perp$ from eigenstructure of Z^{ij}
Radiation reaction coeff. ν	$\nu = \sum_a \gamma_a \, \Xi_a(\mathcal{S})$ (GKSL, Sec. 1.4)
Drag (Rayleigh) Γ	$\Gamma_\eta(\mathcal{S})$ from viscosity channel $\eta > 0$

Table 2.4: Map from spectral couplings to classical observables (schematic).

Chapter 3

Mass–Energy Equivalence as a Spectral Theorem

3.1 Definitions: Energy, Momentum, and the Emergent Light Cone

Quadratic sector and principal symbol. Around an ordered background S_0 (Vol. I, Ch. 2) small fluctuations $\delta\mathbf{S} \equiv (\delta\rho, \delta\eta, \delta\alpha, \delta\chi_I, \delta\Phi, \ldots)^\top$ are governed to leading order by

$$\mathcal{L}^{(2)} = \tfrac{1}{2}\,\delta\mathbf{S}^\top \Big[\mathsf{K}\,\partial_t^2 + \mathsf{C}\,\partial_t + \mathsf{Z}^{ij}\nabla_i\nabla_j + \mathsf{M}^2\Big]\delta\mathbf{S}, \tag{3.1}$$

with $\mathsf{K} \succ 0$ (no ghosts), Z^{ij} positive on spatial covectors (hyperbolicity), and the anti-Hermitian block $\mathsf{C} \propto \eta$ encoding Rayleigh–type dissipation (Vol. II, Sec. **??**). The *principal symbol* for a space–time covector $\xi_\mu = (\xi_0, \xi_i)$ is

$$\mathscr{P}(\xi) = \xi_0^2\,\mathsf{K} + \xi_i\xi_j\,\mathsf{Z}^{ij}, \qquad \text{and the characteristic set solves} \quad \det\mathscr{P}(\xi) = 0, \tag{3.2}$$

where dissipative and mass terms are subleading in the high-frequency limit.

Emergent cone and the universal speed c. For a unit spatial direction $\hat{\mathbf{n}}$ in the aether frame u^μ, define the directional matrix $\mathsf{V}(\hat{\mathbf{n}}) \equiv \mathsf{K}^{-1}\,\mathsf{Z}^{ij}\hat{n}_i\hat{n}_j$. Let $\lambda_{\max}(\hat{\mathbf{n}})$ be its largest eigenvalue. The *front velocity* (light-cone slope) in that direction is

$$c^2(\hat{\mathbf{n}}) = \lambda_{\max}\Big(\mathsf{V}(\hat{\mathbf{n}})\Big), \qquad 0 < c(\hat{\mathbf{n}}) \le c, \tag{3.3}$$

and the *universal limiting speed* is the envelope

$$\boxed{c^2 \equiv \sup_{\hat{\mathbf{n}}} \lambda_{\max}\Big(\mathsf{K}^{-1}\mathsf{Z}^{ij}\hat{n}_i\hat{n}_j\Big)}. \tag{3.4}$$

In the isotropic restoration limit (small anisotropy relative to the director $\hat{\chi}$), $\mathsf{Z}^{ij} = Z\,\delta^{ij}$ for the mode under study and $c^2 = Z/K$.

Canonical momenta and energy density. The canonical momentum conjugate to $\delta\mathbf{S}$ is

$$\delta\mathbf{\Pi} \equiv \frac{\partial\mathcal{L}^{(2)}}{\partial(\partial_t\delta\mathbf{S})} = \mathsf{K}\,\partial_t\delta\mathbf{S} + \tfrac{1}{2}\,\mathsf{C}\,\delta\mathbf{S}. \tag{3.5}$$

Up to small dissipative corrections (which appear as sink terms but do not modify the conservative quadratic form), the Hamiltonian density is

$$\mathcal{E} = \tfrac{1}{2}\Big[(\partial_t\delta\mathbf{S})^\top \mathsf{K}\,(\partial_t\delta\mathbf{S}) + (\nabla_i\delta\mathbf{S})^\top \mathsf{Z}^{ij}(\nabla_j\delta\mathbf{S}) + \delta\mathbf{S}^\top \mathsf{M}^2\delta\mathbf{S}\Big], \tag{3.6}$$

and the momentum density is

$$\mathcal{P}_i \equiv (\partial_t\delta\mathbf{S})^\top \mathsf{Z}^{ij}\nabla_j\delta\mathbf{S}. \tag{3.7}$$

Total energy and momentum in a Cauchy slice Σ_t are

$$E = \int_{\Sigma_t} d^3x\,\mathcal{E}, \qquad p_i = \int_{\Sigma_t} d^3x\,\mathcal{P}_i. \tag{3.8}$$

Energy flux and continuity. Let $\mathcal{F}^i \equiv -(\partial_t\delta\mathbf{S})^\top \mathsf{Z}^{ij}\partial_t\nabla_j\delta\mathbf{S}$ be the conservative energy flux (the Poynting analogue). Using the equations of motion from (3.1) one obtains the balance law

$$\partial_t\mathcal{E} + \nabla_i\mathcal{F}^i \;=\; -\,\sigma_{\rm diss}, \qquad \sigma_{\rm diss} \;\equiv\; \tfrac{1}{2}\,(\partial_t\delta\mathbf{S})^\top\Big(\mathsf{C}^\top\mathsf{K}^{-1}\mathsf{C}\Big)(\partial_t\delta\mathbf{S}) \;\geq 0, \tag{3.9}$$

so dissipation ($\eta > 0$) monotonically removes energy from the quadratic sector, consistent with the entropy gate (Vol. I).

Four–momentum and invariant mass. In the local orthonormal frame of the emergent metric $g_{\rm eff}$, define the four–momentum

$$P^\mu \equiv \left(\frac{E}{c},\,\mathbf{p}\right), \qquad \mathbf{p} \equiv (p^1, p^2, p^3). \tag{3.10}$$

Isometries of $g_{\rm eff}$ act as local Lorentz transformations on P^μ. The *spectral rest mass* of a (possibly composite/solitonic) excitation is

$$\boxed{m_{\rm eff}^2c^2 \equiv g^{\rm eff}_{\mu\nu}\,P^\mu P^\nu = \frac{E^2}{c^2} - \mathbf{p}^{\,2}}. \tag{3.11}$$

For localized, stationary configurations ($\mathbf{p} = 0$) this reduces to $m_{\rm eff} = E/c^2$. For weakly anisotropic media one may define directional $\mathbf{p}$-norms using the quadratic form induced by $\mathsf{K}^{-1}\mathsf{Z}^{ij}$, still bounded by the universal c in (3.4).

Group, phase, and front velocities. Normal modes with dispersion $\omega(\mathbf{k})$ have $\mathbf{v}_{\rm ph} = \omega\,\mathbf{k}/k^2$, $\mathbf{v}_{\rm g} = \nabla_{\mathbf{k}}\omega$, and the *front* speed $v_{\rm fr}(\hat{\mathbf{k}}) = \lim_{k\to\infty}\omega/k = c(\hat{\mathbf{k}})$ by (3.3). Consistency gates require[1]

$$0 < c_s(\hat{\mathbf{k}}),\, c_T(\hat{\mathbf{k}}) \leq c(\hat{\mathbf{k}}) \leq c, \qquad \text{and} \quad \mathsf{K} \succ 0,\; \mathsf{Z}^{ij} > 0. \tag{3.12}$$

[1]To avoid Cerenkov–type instabilities and retain hyperbolicity: $0 < v_{\rm g} \leq c(\hat{\mathbf{k}}) \leq c$, with c given by (3.4).

Remarks on anisotropy. If $\hat{\chi}$ selects a privileged direction, Z^{ij} splits into longitudinal/transverse blocks: $Z_{\parallel}\,\hat{\chi}^i\hat{\chi}^j + Z_{\perp}(\delta^{ij} - \hat{\chi}^i\hat{\chi}^j)$. Then $c^2_{\parallel,\perp} = Z_{\parallel,\perp}/K$ and observations demand $|c_{\parallel} - c_{\perp}| \ll c$ in regimes where near–isotropy is established (Vol. II, Sec. **??**).

The definitions above fix the energetic and kinematic ledger used in the rest of the chapter: the universal speed c from the principal symbol, the Noether charges $(E, \mathbf{p})$, and the invariant spectral mass m_{eff} as in (3.11). In Sec. 3.2 we show that these data imply the mass–energy relation $E^2 = (pc)^2 + (m_{\mathrm{eff}}c^2)^2$ and recover $E_0 = m_{\mathrm{eff}}c^2$ for stationary configurations.

3.2 Dispersion and the Mass–Energy Theorem

Normal form and single–branch dispersion

Start from the quadratic action (3.1). There exists a real, invertible field redefinition[2] $\delta\Psi \equiv \mathsf{R}^{-1}\delta\mathbf{S}$ with

$$\mathcal{L}^{(2)} = \tfrac{1}{2}\,\delta\Psi^{\top}\Big[-\partial_t^2 + \mathsf{A}^{ij}\nabla_i\nabla_j + (\mathsf{M}_-^*)^2\Big]\,\delta\Psi \;+\; \mathcal{O}(\eta), \qquad \mathsf{A}^{ij} \equiv \mathsf{R}^{\top}\mathsf{Z}^{ij}\mathsf{R}, \quad (\mathsf{M}_-^*)^2 \equiv \mathsf{R}^{\top}\mathsf{M}^2\mathsf{R}. \tag{3.13}$$

(The small anti-Hermitian block C only supplies $\mathcal{O}(\eta)$ dissipative corrections and will be omitted until Sec. 3.2.) For a plane-wave ansatz $\propto e^{i(\mathbf{k}\mathbf{x}-\omega t)}$ the dispersion relation is the generalized eigenvalue problem

$$\det\Big[-\omega^2\mathbb{I} + \mathsf{A}^{ij}k_ik_j + \big(\mathsf{M}_-^*\big)^2\Big] = 0. \tag{3.14}$$

Let α index a propagating branch (normal mode). For $|\mathbf{k}|$ below the cutoff Λ_{sc} the lowest two orders read

$$\omega_\alpha^2(\mathbf{k}) \;=\; c_\alpha^2(\hat{\mathbf{k}})\,k^2 + \omega_{\alpha,0}^2 + \mathcal{O}\Big(\tfrac{k^4}{\Lambda_{\mathrm{sc}}^2}\Big), \qquad c_\alpha^2(\hat{\mathbf{k}}) = \lambda_{\max}\Big(\mathsf{P}_\alpha^{\top}\,\mathsf{A}^{ij}\,\hat{k}_i\hat{k}_j\,\mathsf{P}_\alpha\Big), \qquad \omega_{\alpha,0}^2 = \lambda_{\min}\Big(\mathsf{P}_\alpha^{\top}\,\mathsf{M}\,\mathsf{P}_\alpha\Big). \tag{3.15}$$

with P_α the projector on the α–subspace (after diagonalization at $\mathbf{k} = 0$). Hyperbolicity implies $0 < c_\alpha(\hat{\mathbf{k}}) \le c$ where c is the universal front speed defined in (3.4). The gap $\omega_{\alpha,0}$ is the *rest frequency* (mass gap) of branch α.

Hamilton–Jacobi form and the effective mass shell

Introduce the eikonal phase $S(x)$ and four–momentum $p_\mu \equiv \partial_\mu S = (-\omega, \mathbf{k})$. Along branch α the characteristic polynomial (3.14) induces a quadratic mass–shell (up to k^4 corrections):

$$g_{\mathrm{eff}}^{\mu\nu}p_\mu p_\nu \;+\; m_\alpha^2c^2 \;=\; \mathcal{O}\!\left(\frac{k^4}{\Lambda_{\mathrm{sc}}^2}\right), \qquad m_\alpha c^2 \equiv \hbar\,\omega_{\alpha,0}\,, \tag{3.16}$$

where $g_{\mathrm{eff}}^{\mu\nu}$ is the (locally isotropized) inverse emergent metric determined by the principal symbol and we reinstated $\hbar$ for later quantum use.[3] In components (local orthonormal

[2] A symmetric (Friedrichs) symmetrizer; choose R such that $\mathsf{R}^{\top}\mathsf{K}\mathsf{R} = \mathbb{I}$.

[3] Classically one may set $\hbar = 1$; the identification marries the spectral gap to the rest energy.

frame of g_{eff})

$$\omega_\alpha^2 \;=\; c^2k^2 \;+\; \omega_{\alpha,0}^2 \;+\; \mathcal{O}(k^4/\Lambda_{\text{sc}}^2) \quad\Longleftrightarrow\quad \frac{E^2}{c^2} - \mathbf{p}^{\,2} \;=\; m_\alpha^2 c^2 \;+\; \mathcal{O}\!\left(\frac{p^4}{\Lambda_{\text{sc}}^2 c^2}\right), \tag{3.17}$$

upon the Noether identification $(E, \mathbf{p}) = (\hbar\omega, \hbar\,\mathbf{k})$.

Theorem 3.1 (Mass–energy equivalence for a spectral branch)**.** *Consider a localized, finite–energy wavepacket entirely supported on a single stable branch α of (3.14), in a region where (i) $\mathsf{K} \succ 0$, $\mathsf{Z}^{ij} > 0$, (ii) the front velocity is bounded by c, and (iii) the quartic corrections are negligible ($p \ll \Lambda_{\text{sc}}\, c$). Then its Noether four–momentum $P^\mu = (E/c, \mathbf{p})$ satisfies the relativistic mass shell*

$$E^2 \;=\; (pc)^2 \;+\; (m_\alpha c^2)^2 \qquad \textit{with} \qquad m_\alpha c^2 = \int_{\Sigma_t} d^3x\, \mathcal{E}_{\text{rest}} \;=\; E_0, \tag{3.18}$$

where $\mathcal{E}_{\text{rest}}$ is the energy density (3.6) of the stationary configuration and E_0 its total (rest) energy.

Sketch. (i) Diagonalize the quadratic Hamiltonian using (3.13); the single–branch packet is a superposition of modes with the same dispersion $\omega_\alpha(\mathbf{k})$. (ii) The group velocity $\mathbf{v}_{\text{g}} = \partial_{\mathbf{k}}\omega_\alpha$ is subluminal due to (3.12) and convexity of $\omega_\alpha(\mathbf{k})$. (iii) Define $E(\mathbf{p}) \equiv \hbar\omega_\alpha(\mathbf{k})$ with $\mathbf{p} = \hbar\mathbf{k}$. The function $E(\mathbf{p})$ is even, strictly convex, and $E(0) = E_0 = \hbar\omega_{\alpha,0}$. By rotational symmetry in the isotropized frame, $E(\mathbf{p}) = f(p)$ with $f'(0) = 0$ and $f''(0) = 1/m_\alpha$, where $m_\alpha \equiv E_0/c^2$. The *front* bound, $\lim_{p\to\infty} dE/dp \le c$, integrates to $E^2 - (pc)^2 \ge E_0^2$; saturation at small and moderate p follows from the quadratic approximation (3.17), yielding $E^2 = (pc)^2 + E_0^2$ throughout the validity domain ($p \ll \Lambda_{\text{sc}}c$). □

Rest energy, kinetic expansion, and equality of masses

Equation (3.18) immediately gives the rest energy

$$\boxed{E_0 \;=\; m_\alpha c^2}, \tag{3.19}$$

with m_α computable as the integral of the quadratic density in the static soliton/packet core or, in the particle sector, as the spectral mass of the excitation. For small momenta, expand

$$E(p) = m_\alpha c^2 + \frac{p^2}{2m_\alpha} - \frac{p^4}{8m_\alpha^3 c^2} + \mathcal{O}\!\left(p^6/(m_\alpha^5 c^4)\right), \tag{3.20}$$

so the *inertial* mass (coefficient of $p^2/2$) equals the *rest* mass and, by the arguments of Vol. III, Sec. **??**, the *gravitational* mass that couples to g_{eff}—an *Equivalence Gate* in this framework.

Anisotropy and controlled deviations

When a residual director $\hat{\chi}$ is present, the quadratic norm of momentum is mildly anisotropic:

$$\mathbf{p}^{\,2} \;\longrightarrow\; \mathbf{p}^{\top}\!\Big[c_\parallel^{-2}\,\hat{\chi}\hat{\chi}^{\top} + c_\perp^{-2}\,(\mathbb{I} - \hat{\chi}\hat{\chi}^{\top})\Big]\mathbf{p} \;=\; \frac{p_\parallel^2}{c_\parallel^2} + \frac{p_\perp^2}{c_\perp^2}, \tag{3.21}$$

with $|c_\parallel - c_\perp| \ll c$ during the restored phase. The mass–shell is

$$E^2 \;=\; (m_\alpha c^2)^2 \;+\; c^2\!\left(\frac{p_\parallel^2}{1+\delta_\parallel} + \frac{p_\perp^2}{1+\delta_\perp}\right), \qquad \delta_{\parallel,\perp} \equiv 1 - \frac{c_{\parallel,\perp}^2}{c^2}\,, \tag{3.22}$$

whose angle–averaged form reproduces (3.18). Observational bounds (CMB/LSS, Vol. II, Sec. **??**) require $|\delta_{\parallel,\perp}| \lesssim 10^{-2}$–$10^{-3}$ in the early universe and much tighter today.

Packets, boosts, and additivity

Let $\mathcal{P}$ be the packet's 3–momentum and $\mathbf{v} \equiv dE/d\mathcal{P}$ its group velocity. With (3.18) one finds

$$\mathbf{v} = \frac{c^2\mathbf{p}}{E}, \qquad \gamma \equiv \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{E}{m_\alpha c^2}, \qquad \mathbf{p} = \gamma m_\alpha \mathbf{v}, \tag{3.23}$$

matching the standard velocity–momentum map. For weakly interacting packets, Noether charges add linearly, and the total four–momentum transforms as a g_{eff}–four–vector, so the invariant mass of a composite is obtained from $(\sum P^\mu)(\sum P_\mu)$, reproducing the usual relativistic composition rules.

Dissipative corrections and validity domain

Reinstating C, the dispersion acquires an imaginary part, $\omega \to \omega - i\Gamma/2$ with $\Gamma = \mathcal{O}(\eta)$, and the energy balance (3.9) shows $\dot{E} = -\int d^3x\, \sigma_{\mathrm{diss}} \le 0$. For processes on timescales $\tau \ll \Gamma^{-1}$ the conservative dispersion and (3.18) remain accurate. Beyond that, one separates *quasiparticle* (underdamped) from *diffusive* (overdamped) regimes; the mass–energy relation applies to the former, while the latter do not admit a particle interpretation and are treated hydrodynamically.

Summary. Under the spectral consistency gates (positivity, hyperbolicity, subluminal front speed) each stable branch admits a Lorentz–type mass shell built from the emergent metric g_{eff}. The rest energy equals mc^2, the moving energy obeys $E^2 = (pc)^2 + (mc^2)^2$ up to controlled $\mathcal{O}(p^4/\Lambda_{\mathrm{sc}}^2)$ and tiny anisotropy corrections, and the standard kinematic identities follow. The constant c is not postulated: it is the supremum of principal–symbol phase speeds (3.4), i.e. a *spectral property* of the medium.

3.3 Rest Mass of Solitons and Binding Energies

Finite–energy solitons and the rest functional

A (static) soliton is a finite–energy stationary solution $\mathcal{S}_{\mathrm{sol}}(\mathbf{x};\mathcal{M})$ to the Euler–Lagrange equations of the conservative sector (Vol. I, Ch. 4), labelled by collective coordinates (moduli) $\mathcal{M} = \{X^i, \Theta^A\}$ (center, orientation, size, phase, …). With $\eta > 0$ the explicit Rayleigh term vanishes for static profiles, so the rest energy is purely conservative:

$$E_0[\mathcal{S}_{\mathrm{sol}}] \;=\; \int d^3x\, \mathcal{H}\big(\mathcal{S}_{\mathrm{sol}}, \nabla\mathcal{S}_{\mathrm{sol}}\big), \qquad M_{\mathrm{sol}} \;\equiv\; \frac{E_0}{c^2}\,. \tag{3.24}$$

Here $\mathcal{H}$ is the T^{00}_{eff} density associated with the quadratic+quartic structural Lagrangian and matter sector (Vol. I, Ch. 3–4). Boundary conditions are chosen so that $\mathcal{S}_{\mathrm{sol}} \to \mathcal{S}_0$ fast enough and topological charges (if any) are well defined.

Collective coordinates and the moduli metric

Let $\psi_A(\mathbf{x}) \equiv \partial_{\Theta^A}\mathcal{S}_{\rm sol}$ denote zero modes and $\psi_i \equiv \partial_{X^i}\mathcal{S}_{\rm sol}$ the translational modes. Upon a slow time dependence of moduli, the quadratic action (3.1) induces an effective particle Lagrangian

$$L_{\rm eff}(\dot{\mathcal{M}}) \;=\; \frac{1}{2}\,G_{AB}\,\dot\Theta^A\dot\Theta^B + \frac{1}{2}\,M_{\rm sol}\,\dot{\mathbf{X}}^2 \;-\; U_{\rm int}(\Theta) \;+\; \mathcal{O}(\dot{\mathcal{M}}^4), \tag{3.25}$$

with the *moduli metric*

$$G_{AB} \;=\; \int d^3x\; \psi_A^\top\,\mathsf{K}\,\psi_B\,, \qquad M_{\rm sol}\,\delta_{ij} \;=\; \int d^3x\; \psi_i^\top\,\mathsf{K}\,\psi_j\,, \tag{3.26}$$

where K is the positive time–kinetic block (Sec. 3.1). Equation (3.26) shows that the *translational* block equals $M_{\rm sol}\delta_{ij}$, ensuring consistency with the dispersion result $E \simeq M_{\rm sol}c^2 + \frac{p^2}{2M_{\rm sol}}$ (Sec. 3.2). The potential $U_{\rm int}$ collects *intrinsic* elastic energies (e.g. core deformations) in the soliton frame.

Virial/Derrick identities. For scale transformations $\mathbf{x} \mapsto \lambda\mathbf{x}$, $\mathcal{S} \mapsto \mathcal{S}_\lambda$, stationarity of $E_0[\mathcal{S}_\lambda]$ at $\lambda = 1$ yields a generalized Derrick identity relating gradient and potential energies; this provides a *nontrivial check* of numerics and a closure condition for analytic ansätze used to estimate $M_{\rm sol}$.

Boosted soliton and equality of masses

Let $\mathcal{S}_{\rm sol}(\mathbf{x})$ be static in its rest frame. The slowly boosted profile $\mathcal{S}_v(\mathbf{x},t) \approx \mathcal{S}_{\rm sol}(\mathbf{x} - \mathbf{v}t)$ gives, to quadratic order in v,

$$E(v) \;=\; M_{\rm sol}c^2 + \frac{1}{2}M_{\rm sol}v^2 + \mathcal{O}\!\left(v^4/c^2\right), \qquad \mathbf{P} \;=\; M_{\rm sol}\mathbf{v} + \mathcal{O}\!\left(v^3/c^2\right), \tag{3.27}$$

so the *inertial* mass equals the rest mass defined by (3.24). Coupling to $g_{\rm eff}$ (Vol. III, Sec. **??**) then shows the *gravitational* mass equals $M_{\rm sol}$ as well, within the restoration gate (small anisotropy, weak dissipation).

Topological lower bounds and scaling with charge

For sectors with a conserved topological charge $Q_{\rm top}$ one often has inequalities of the form

$$E_0 \;\geq\; \mathcal{B}\left|Q_{\rm top}\right|^{\sigma}, \qquad \sigma \in \{1, 3/4, \dots\}, \tag{3.28}$$

e.g. linear for strings/walls, $\sigma = 3/4$ for Hopfions in Faddeev–Skyrme–type energies (Vol. I, Ch. 4). When a near–BPS completion exists, multi–charge solutions may *saturate* or nearly saturate the bound, informing the sign and magnitude of binding energies for composites at fixed $Q_{\rm top}$.

Binding energies of composites

Consider a composite configuration $\mathcal{C}$ made of well–separated solitons $\{\mathcal{S}_a\}_{a=1}^N$ with centers $\mathbf{X}_a$, relative orientations Θ_a, and separations R_{ab}. Its energy admits the cluster expansion

$$E_0[\mathcal{C}] \;=\; \sum_{a=1}^{N} E_0[\mathcal{S}_a] \;+\; \sum_{a<b} V_{ab}(R_{ab}, \Theta_a, \Theta_b; \hat\chi) \;+\; \mathcal{O}\!\left(e^{-m_* R}/R^2\right), \tag{3.29}$$

where V_{ab} arises from linear exchange of massive spectral modes and from elastic overlap of cores. For large R one typically has Yukawa/logarithmic tails,

$$V_{ab}(R,\Theta;\hat{\chi}) \;\simeq\; -\sum_I \frac{\mathcal{Q}_a^{(I)}\,\mathcal{Q}_b^{(I)}}{4\pi}\,\frac{e^{-m_I(\Theta,\hat{\chi})R}}{R}\;\times\;\Big[1+\mathcal{A}^{(I)}(\Theta,\hat{\chi})\Big], \tag{3.30}$$

with $\mathcal{Q}^{(I)}$ the relevant spectral charges/couplings and m_I the inverse correlation lengths of the exchanged channels; the angular factor encodes anisotropy relative to the internal director $\hat{\chi}$. The *binding energy* and the *mass defect* of the composite are

$$E_{\rm bind} \;\equiv\; E_0[\mathcal{C}] - \sum_a E_0[\mathcal{S}_a] \;\le\; 0, \qquad \Delta m \;\equiv\; \sum_a M_{{\rm sol},a} - M_{\rm bound} \;=\; \frac{|E_{\rm bind}|}{c^2}. \tag{3.31}$$

A negative $E_{\rm bind}$ signals a stable bound state at the given quantum numbers ($Q_{\rm top}$, Noether charges); metastability may occur if there are lower channels separated by barriers in moduli space.

Moduli–space dynamics and capture. For two solitons at large R,

$$L_{\rm rel} = \frac{1}{2}\mu\,\dot{R}^2 + \frac{1}{2}\mathcal{I}(\Theta)\,\dot{\Theta}^2 - V_{12}(R,\Theta;\hat{\chi}), \qquad \mu = \frac{M_1 M_2}{M_1+M_2}, \tag{3.32}$$

plus small dissipative terms $\propto \eta$ (Sec. 3.2). Adiabatic radiation into gapped spectral channels leads to a slow decrease of the Jacobi constant and *capture* into a bound orbit when the pericenter samples the attractive core of V_{12}; the corresponding energy loss sets the timescale of composite formation.

Canonical examples (benchmarks)

Straight vortices (line defects). For winding n, energy per unit length $\mathcal{T}_n$ (Vol. I, Ch. 4) and interaction $V_{12}/L \simeq -\kappa\, n_1 n_2\, K_0(m_{\rm eff}R)$ with K_0 a modified Bessel and $m_{\rm eff}$ the mass of the screened circulation; parallel/antiparallel windings attract/repel. The mass defect of a bound pair of length L is $|E_{\rm bind}|/c^2 \sim \kappa|n_1 n_2|L$ up to core corrections.

Domain walls. Two parallel walls of tension σ interact via $V_{12} \sim \pm\sigma_{\rm eff}e^{-m_\rho R}$ (repulsive/attractive depending on relative phase of the order parameter), yielding composite "double–wall" states with reduced effective tension and $\Delta m \propto \sigma_{\rm eff}A/c^2$.

Hopfions/Skyrmions. For Hopf charge Q_H, $E_0 \propto |Q_H|^{3/4}$ gives a subadditive scaling, so multi–Hopf composites are *energetically favored* at fixed Q_H, with binding fraction $1 - E(Q_1+Q_2)/(E(Q_1)+E(Q_2))$ controlled by the quartic stiffness and curvature terms of the Faddeev–Skyrme sector.

Stress balance and virial test for composites

Let $T^{ij}_{\rm eff}$ be the spatial stress of the conservative theory. For any static, finite–energy configuration in $\mathbb{R}^3$ one has

$$\int d^3x\, T^i{}_i \;=\; 0 \qquad\Longleftrightarrow\qquad \sum_{\rm grad}\alpha_r\,E_r \;=\; \sum_{\rm pot}\beta_s\,E_s, \tag{3.33}$$

with α_r, β_s the scaling weights of gradient/potential pieces. Equation (3.33) furnishes a stringent consistency check for numerical composites and an analytic handle to bracket $E_{\rm bind}$.

Dissipative corrections and mass renormalization

Small but finite η induces a frequency–dependent self–energy $\Sigma(\omega)$ for normal modes; to leading order the *static* ($\omega = 0$) rest mass is unchanged, while the inertial response acquires a tiny (positive) renormalization:

$$M_{\rm sol}^{(\rm inertial)} \;=\; M_{\rm sol}\Big[1 + \delta_{\rm in}(\eta, \chi^\mu)\Big], \qquad \delta_{\rm in} = \mathcal{O}\Big(\frac{\eta}{\Lambda_{\rm sc}}\Big)\,, \tag{3.34}$$

and slow secular relaxation of excited composites lowers E_0 toward the nearest static minimum, increasing $|E_{\rm bind}|$ by $\mathcal{O}(\eta\,\Delta t)$ until a stationary bound state is reached.

Summary. Soliton rest masses are integrals of the conservative energy density (3.24); their inertial and gravitational masses coincide under the spectral gates. Composite objects exhibit binding when the exchange/overlap potential is negative, with a mass defect $\Delta m = |E_{\rm bind}|/c^2$. Topological lower bounds, virial identities, and moduli–space dynamics provide a complete toolkit to *compute, bound, and validate* $M_{\rm sol}$ and $E_{\rm bind}$ across sectors (walls, vortices, Q-balls, Hopfions) within the Quantum–Aether framework.

3.4 Universality of c: RG Flow and Constraints

Objective. We show that, under coarse–graining and weak dissipation ($\eta > 0$), the *propagation speeds* of all linear sectors (scalar Φ, structural modes in ρ, χ^μ, T, and tensor perturbations of $g_{\rm eff}$) *collapse* to a common infrared (IR) value c_*. This *universal front speed* provides the operational constant in the mass–energy relation of Secs. 3.1–3.3. We then map observational bounds on inter-sector speed differences to constraints on spectral couplings.

Sectoral speeds and anisotropy parameters

For a linearized mode φ_A (index A labels a diagonalized sector) the quadratic Lagrangian density in the aether frame u^μ reads

$$\mathcal{L}_A^{(2)} = \frac{K_A}{2}\,\dot\varphi_A^{\,2} - \frac{1}{2}\Big(Z_{A,\parallel}\,(\nabla_\parallel\varphi_A)^2 + Z_{A,\perp}\,(\nabla_\perp\varphi_A)^2\Big) - \frac{1}{2}\,m_{A,\rm eff}^2\,\varphi_A^2 - \frac{\Gamma_A}{2}\,\dot\varphi_A^{\,2}, \qquad K_A > 0, \tag{3.35}$$

with $\nabla_\parallel \equiv \hat\chi^{(0)}\cdot\nabla$ and $\nabla_\perp$ orthogonal to $\hat\chi^{(0)}$. Phase speeds at low damping are

$$c_{A,\parallel}^2 \equiv \frac{Z_{A,\parallel}}{K_A}, \qquad c_{A,\perp}^2 \equiv \frac{Z_{A,\perp}}{K_A}, \qquad \Delta_A \equiv c_{A,\parallel}^2 - c_{A,\perp}^2. \tag{3.36}$$

Tensor (gravitational) waves on $g_{\rm eff}$ satisfy

$$\mathcal{L}_T^{(2)} = \frac{M_{\rm eff}^2}{8}\,\dot h_{ij}^2 - \frac{M_{\rm eff}^2\,c_T^2}{8}\,(\partial_k h_{ij})^2 + \cdots, \qquad c_T^2 = 1 + \delta c_T(\mathcal{S}), \tag{3.37}$$

with small corrections δc_T from higher-curvature spectral terms (Vol. I, Ch. 3; Vol. II, Sec. **??**).

RG flow to a universal cone

Let ℓ be the coarse–graining scale and $b \equiv \ln(\ell/\ell_0)$ the RG time. Write the minimal running system

$$\frac{d}{db}\begin{pmatrix} Z_{A,\perp} \\ \Delta_A \\ K_A \end{pmatrix} = \begin{pmatrix} \beta_{Z_\perp} \\ \beta_\Delta \\ \beta_K \end{pmatrix} (Z_{A,\perp}, \Delta_A, K_A; \eta, \rho, \ldots), \qquad \beta_\Delta = -\alpha_\Delta\, \Delta_A + \mathcal{O}(\Delta_A^2), \quad \alpha_\Delta > 0, \tag{3.38}$$

where $\alpha_\Delta > 0$ is generated by angular averaging and weak dissipative mixing ($\eta > 0$) that suppress anisotropic gradient operators.[4] Then $\Delta_A \to 0$ exponentially in b, and ratios of sectoral speeds obey

$$\frac{d}{db}\ln\!\Big(\frac{c_A}{c_B}\Big) = \frac{1}{2}\Big(\frac{\beta_{Z_A}}{Z_A} - \frac{\beta_{K_A}}{K_A} - \frac{\beta_{Z_B}}{Z_B} + \frac{\beta_{K_B}}{K_B}\Big) \xrightarrow[b\to+\infty]{} 0, \tag{3.39}$$

provided the same universality class controls (β_Z, β_K) across sectors. We summarize:

Lemma (Universality Gate). Under (i) no-ghost $K_A > 0$, (ii) hyperbolicity $Z_{A,\perp} > 0$, (iii) weak dissipation $\eta > 0$ and (iv) common IR universality class, all sectoral cones collapse to a *single* IR cone with speed c_*: $c_{A,\parallel}$, $c_{A,\perp}$, $c_T \to c_*$ and $\Delta_A \to 0$.

Front velocity and causality

Let $n_A(\omega, \hat{\mathbf{k}})$ be the refractive index for sector A. Kramers–Kronig plus positivity of absorption imply the *front velocity* $v_{\rm front}^{(A)} = \lim_{\omega\to\infty} \omega/k(\omega)$ equals the high-frequency limit c_∞ of the microphysical cone. The gate **(GQ1)** demands $v_{\rm front}^{(A)} \le c_{\rm eff}^{\rm micro}$ for all A, hence the universal constant entering the mass–energy theorem must be

$$c \;\equiv\; c_* \;=\; c_\infty\,, \tag{3.40}$$

shared by photons, gravitons and all weakly coupled sectors in the IR.

Observational constraints and parameter maps

Differences of arrival times over a baseline D bound inter-sector speed splittings:

$$\left|\frac{c_A - c_B}{c}\right| \;\lesssim\; \frac{\Delta t_{AB}}{D/c}\,. \tag{3.41}$$

Multi-messenger coincidences (GW + EM), high-energy cosmic-ray Cherenkov bounds, and pulsar timing collectively impose $|\delta c_T|/c$, $|\delta c_\gamma|/c \ll 1$ on astrophysical baselines.[5] In the spectral EFT these splittings are generated by higher-order operators:

$$\delta c_T \simeq \frac{\zeta_{\rm curv}}{M_{\rm eff}^2} \;+\; \mathcal{O}\Big(\frac{\nabla\mathcal{S}}{\Lambda^2}\Big), \qquad \delta c_\gamma \simeq \frac{\zeta_{\rm em}}{\Lambda_{\rm em}^2} \;+\; \epsilon_\chi\,(\hat{\mathbf{k}}\cdot\hat{\chi}^{(0)})^2 + \cdots, \tag{3.42}$$

[4] In operator language, anisotropic invariants built from χ^μ (e.g. $\chi^\mu\chi^\nu\nabla_\mu\nabla_\nu$) are *irrelevant* at long wavelengths once $\eta > 0$ opens relaxation channels that smear directional contrasts; see Vol. II, Sec. **??**.

[5] Concrete numerical bounds can be inserted where desired (e.g. binary-neutron-star events constrain $|c_T/c - 1|$ to extremely small values; high-energy cosmic rays bound vacuum-Cherenkov anomalies for photons/electrons). We keep the presentation symbolically to maintain model–space generality.

where $\zeta_{\rm curv}, \zeta_{\rm em}$ are Wilson coefficients of curvature and electromagnetic kinetic-mixing terms inherited from the unified action (Vol. I, Ch. 3), $\Lambda, \Lambda_{\rm em}$ are suppression scales, and ϵ_χ measures residual director anisotropy at the observation scale. Combining Eqs. (3.41)–(3.42) yields priors for inference (cf. Vol. II, Chap. **??**):

$$|\delta c_T|/c \;\le\; \varepsilon_T^{\max}, \qquad |\delta c_\gamma|/c \;\le\; \varepsilon_\gamma^{\max}, \qquad |\epsilon_\chi| \;\le\; \varepsilon_\chi^{\max}, \tag{3.43}$$

with ε's set by the chosen dataset combination.

Corollaries for the mass–energy relation

(i) The constant c appearing in $E = \gamma M c^2$ (Sec. 3.2) is the *front* speed c_* fixed by the IR cone, independent of sector; (ii) permissible deviations δc_A enter only as higher-order, scale-dependent corrections to dispersion and *cannot* alter the universal rest-energy coefficient c^2; (iii) the restoration lemma (Vol. II, Sec. **??**) ensures $c_A \to c$ as $\eta \downarrow 0$ and $\ell \uparrow \infty$.

Summary. Weak dissipation and RG coarse–graining drive all sectoral cones to a universal IR cone with speed c. Causality fixes c to the microphysical front speed; multi-messenger and high-energy observations bound residual splittings and anisotropies, translating directly into limits on spectral Wilson coefficients. This universality underwrites the uniqueness of the c^2 factor in the mass–energy equivalence across the entire spectral medium.

3.5 Energy Balance, Radiation, and GKSL Consistency

Aim. We derive an energy–balance identity on the emergent geometry $g_{\rm eff}$, exhibit the radiative (Poynting) flux and the dissipative production term implied by the spectral medium, and show how a *completely positive* (GKSL) coarse–grained dynamics for the reduced system forbids runaways and pre–acceleration. In the weak–coupling, Markovian limit we recover the standard Larmor law (with effective couplings) for power radiated by an accelerated spectral charge/soliton.

Covariant energy balance on $g_{\rm eff}$

Let $T^{\mu\nu}_{\rm tot} = T^{\mu\nu}_{\rm sys} + T^{\mu\nu}_{\rm rad} + T^{\mu\nu}_{\rm bath}$ be the total stress tensor (system = localized degrees of freedom/solitons, rad = propagating radiation channels, bath = spectral dissipative sector controlled by η). In the absence of external driving, total covariant conservation holds

$$\nabla_\mu T^{\mu\nu}_{\rm tot} = 0, \qquad \nabla \equiv \nabla[g_{\rm eff}]. \tag{3.44}$$

Project on the aether frame u^μ (Vol. I, Ch. 2): define energy density $\varepsilon \equiv T^{\mu\nu}_{\rm tot} u_\mu u_\nu$ and spatial flux $S^\alpha \equiv -h^\alpha{}_\nu T^{\nu\beta} u_\beta$ with $h^\alpha{}_\nu = \delta^\alpha{}_\nu + u^\alpha u_\nu$. In a local inertial frame of u^μ (neglecting $O(\sigma, \omega, H)$ kinematics on subhorizon scales),

$$\partial_t \varepsilon \;+\; \nabla_i S^i \;=\; -\,\mathcal{P}_{\rm work} \;-\; \mathcal{Q}_{\rm diss}\,, \tag{3.45}$$

where

$$\mathcal{P}_{\rm work} \equiv J_\mu\, E^\mu_{\rm eff} \qquad \text{(power from radiative fields on sources)}, \tag{3.46}$$

$$\mathcal{Q}_{\rm diss} \equiv \sigma_{\rm diss} \;\geq 0 \qquad \text{(spectral entropy production; cf. Vol. II, Eq. (2.7))}. \tag{3.47}$$

Here J^μ is the effective source current coupled to the radiative channel (e.g. emergent $U(1)$ or tensor mode), and $E^\mu_{\rm eff}$ the corresponding generalized "electric" field (for an electromagnetic-like sector, $E^i_{\rm eff} = F^{i0}_{\rm eff}$). Equation (3.45) is the Poynting theorem on $g_{\rm eff}$: the change of total energy within a domain equals the outward radiative flux plus nonnegative dissipative production, up to explicit source work.

Radiative sector and generalized Poynting vector

For a gauge-like radiative channel with field strength $F^{\rm eff}_{\mu\nu}$ and Lagrangian $\mathcal{L}_{\rm rad} = -\frac{Z_{\rm em}}{4} F_{\mu\nu}F^{\mu\nu} + J^\mu A_\mu$, the stress tensor $T^{\mu\nu}_{\rm rad} = Z_{\rm em}\Big(F^{\mu\alpha}F^{\nu}{}_{\alpha} - \frac{1}{4} g^{\mu\nu}_{\rm eff} F^2\Big)$ yields, in the aether frame,

$$\varepsilon_{\rm rad} = \tfrac{Z_{\rm em}}{2}\Big(\mathbf{E}^2_{\rm eff} + \mathbf{B}^2_{\rm eff}\Big), \qquad \mathbf{S}_{\rm rad} = Z_{\rm em}\, \mathbf{E}_{\rm eff} \times \mathbf{B}_{\rm eff}. \tag{3.48}$$

Analogous expressions hold for tensor radiation with $Z_{\rm em} \to M^2_{\rm eff}/4$ and $\mathbf{E}_{\rm eff}, \mathbf{B}_{\rm eff}$ replaced by the electric/magnetic parts of the Weyl tensor of $g_{\rm eff}$.

GKSL energy balance and no–runaway

Let ρ be the reduced state of the localized/system sector after tracing out radiative+bath channels in a Markovian window. The spectral derivation of Sec. 1.4 gives a GKSL generator

$$\dot\rho = -i[H_{\rm eff}, \rho] - \sum_a \gamma_a \int d^3x \, \Big(L_a \rho L_a^\dagger - \tfrac{1}{2}\{L_a^\dagger L_a, \rho\}\Big), \qquad \gamma_a(\mathcal{S}) \geq 0. \tag{3.49}$$

The system energy $E_{\rm sys} \equiv \mathrm{Tr}(H_{\rm eff}\rho)$ evolves as

$$\frac{d}{dt} E_{\rm sys} = \mathrm{Tr}\Big((\partial_t H_{\rm eff})\rho\Big) - \sum_a \gamma_a \int d^3x \, \underbrace{\mathrm{Tr}\Big(L_a^\dagger L_a\, \rho - \tfrac{1}{2}\{L_a^\dagger L_a, \rho\}\Big)_{H_{\rm eff}}}_{\Pi_a(\rho) \;\geq 0}, \tag{3.50}$$

where the second term is nonnegative (Spohn's inequality) and equals the *dissipated power* into traced–out channels in the Markovian limit. Combining (3.50) with the field energy balance and integrating (3.45) over a control volume Σ yields the *closed* identity

$$\frac{d}{dt}\Big(E_{\rm sys} + E_{\rm rad}\Big) = -\oint_{\partial\Sigma} \mathbf{S}_{\rm rad}\cdot d\mathbf{\Sigma} \;-\; \int_\Sigma \mathcal{Q}_{\rm diss}\, d^3x \;+\; \mathrm{Tr}\Big((\partial_t H_{\rm eff})\rho\Big), \tag{3.51}$$

which guarantees forward–time well-posedness (contractive semigroup) and *excludes* runaway growth or pre–acceleration pathologies: the GKSL generator cannot generate exponentially increasing $\langle H_{\rm eff}\rangle$ in the absence of explicit driving ($\partial_t H_{\rm eff} = 0$) because the only source terms are nonnegative drains of energy into $\mathbf{S}_{\rm rad}$ and $\mathcal{Q}_{\rm diss}$.

Recovery of Larmor in the weak–coupling limit

Consider a spectral "charge" $q_{\rm eff}$ carried by a small soliton of mass $M_{\rm eff}$ coupled to the emergent $U(1)$. Eliminating the field in the dipole/Markovian limit produces a linear friction kernel and a radiated power

$$P_{\rm rad} = \oint_{\partial\Sigma} \mathbf{S}_{\rm rad}\cdot d\mathbf{\Sigma} = \frac{2}{3}\,\frac{q_{\rm eff}^2}{4\pi\,\varepsilon_{\rm eff}}\,\frac{a^2}{c^3}\,\Big[1+\mathcal{O}(\eta,\ \text{anisotropy})\Big], \tag{3.52}$$

with a the proper acceleration and $\varepsilon_{\rm eff}$ the effective permittivity of the radiative channel (units with c explicit). In the same limit the reduced equation of motion reads

$$M_{\rm eff}\,\dot{\mathbf{v}} = \mathbf{F}_{\rm ext} \;-\; \Gamma_{\rm rad}\,\mathbf{v} \;+\; \boldsymbol{\xi}(t), \qquad \Gamma_{\rm rad} = \frac{2}{3}\,\frac{q_{\rm eff}^2}{4\pi\,\varepsilon_{\rm eff}\,c^3}\,\Omega_c^2, \tag{3.53}$$

where Ω_c is a cutoff set by the Markov window and $\boldsymbol{\xi}$ a zero-mean noise fixed by fluctuation–dissipation. Unlike the Abraham–Lorentz equation, (3.53) is first order in time at the GKSL level, hence exhibits neither pre-acceleration nor runaways. Energy balance closes as $\frac{d}{dt}(\frac{1}{2}M_{\rm eff}v^2) = \mathbf{F}_{\rm ext}\cdot\mathbf{v} - \Gamma_{\rm rad}v^2$, with $\Gamma_{\rm rad}v^2$ matching (3.52) upon averaging over orbital timescales.

Consistency gates and observational priors

The identity (3.51) is conditioned on the gates: (i) no ghosts $K_A > 0$; (ii) hyperbolicity and subluminality of the principal symbol (energy flux causal with respect to $g_{\rm eff}$); (iii) $\mathcal{Q}_{\rm diss} \geq 0$ (spectral entropy production; Vol. II, Sec. **??**); (iv) restoration in the IR so that radiative cones share the universal speed c (Sec. 3.4). Multi–messenger constraints on $|\delta c_T|/c$ and high–energy bounds on vacuum Cherenkov losses translate into limits on $Z_{\rm em}$, $q_{\rm eff}$, and the Wilson coefficients that enter (3.48)–(3.52), providing direct priors for inference pipelines (Vol. II, Chap. **??**).

Summary. On the emergent geometry $g_{\rm eff}$, the spectral Poynting theorem (3.45) and the GKSL balance (3.50) combine into the master identity (3.51), ensuring causal energy flow, nonnegative dissipation, and the absence of runaway pathologies. In the weak–coupling limit the radiated power reduces to the Larmor form with effective couplings, fixing the link between radiation reaction, complete positivity, and the universal c entering the mass–energy relation.

3.6 Tests, Calibrations, and Numerical Showcases

Aim. We collect a set of *operational* tests that (i) calibrate the mass–energy map $E = M_{\rm eff}c^2$ (Secs. 3.1–3.4), (ii) validate energy balance and radiation reaction (Sec. 3.5), and (iii) provide reproducible numerical demonstrations. Each item specifies the measured quantity, the theoretical target, and a pass/fail tolerance compatible with the consistency gates.

3.6.1 Compton calibration and annihilation calorimetry

Spectral Compton frequency. For a localized, approximately spherically symmetric soliton at rest, the quadratic spectral action (Sec. 1.3) fixes the small oscillations around

the static profile $S_\star$ with dispersion

$$\omega^2(\mathbf{k}) = c^2k^2 + \omega_0^2 + \mathcal{O}(k^4), \qquad \omega_0 \equiv \frac{M_{\rm eff}c^2}{\hbar}, \tag{3.54}$$

where c is the common front speed in the IR (Sec. 3.4) and $\hbar$ is the quantization scale extracted from the symplectic form (Sec. 1.3). Measuring the *rest* oscillation frequency ω_0 (autocorrelation peak of the collective coordinate) calibrates $M_{\rm eff} = \hbar\omega_0/c^2$ independently of any boost.

Soliton–antisoliton annihilation. Prepare a well-separated soliton/antisoliton pair with equal rest mass $M_{\rm eff}$ and negligible initial kinetic energy. Evolve with absorbing boundaries and integrate the radiative Poynting flux (Sec. 3.5) over a sphere $\partial\Sigma$:

$$E_{\rm rad} \equiv \int dt \oint_{\partial\Sigma} \mathbf{S}_{\rm rad}\cdot d\mathbf{\Sigma}. \tag{3.55}$$

Energy accounting (GKSL + Poynting identity (3.51)) implies $E_{\rm rad} = 2M_{\rm eff}c^2 - E^{\rm in}_{\rm bind} - E^{\rm resid}_{\rm bath}$, where $E^{\rm in}_{\rm bind} \le 0$ is any initial binding and $E^{\rm resid}_{\rm bath} \ge 0$ is the net heat lost to dissipative channels. A *calorimetric pass* requires

$$\frac{\left|E_{\rm rad} + E^{\rm resid}_{\rm bath} - 2M_{\rm eff}c^2 + E^{\rm in}_{\rm bind}\right|}{2M_{\rm eff}c^2} \;\le\; 10^{-3}, \tag{3.56}$$

with the budget split reported explicitly.

3.6.2 Nuclear-scale binding as a mass defect

Consider a composite N-soliton bound state with static fields $S_{(N)}$. Let the energy functional (Vol. I, Ch. 3) be

$$E[S] = \int d^3x \,\Big(\mathcal{K}[S;\partial S] + \mathcal{V}[S] + \mathcal{C}[S;\omega_\mu]\Big), \tag{3.57}$$

where $\mathcal{K}$ (quadratic gradients), $\mathcal{V}$ (potential), and $\mathcal{C}$ (curvature/Skyrme–like) terms are positive in the allowed region. Define the composite rest mass and the *mass defect*

$$M_{(N)} \equiv \frac{E[S_{(N)}]}{c^2}, \qquad \Delta m \equiv N\,M_{\rm eff} - M_{(N)} = \frac{|E_{\rm bind}|}{c^2}, \quad E_{\rm bind} \le 0. \tag{3.58}$$

A successful binding calibration must satisfy: (i) virial/stationarity checks for $S_{(N)}$, (ii) hyperbolicity/no-ghost gates for fluctuations about $S_{(N)}$, and (iii) a percent-level closure of the energy ledger between $\sum_i E[S^{(i)}_\star]$ and $E[S_{(N)}] + |E_{\rm bind}|$.

3.6.3 Numerical showcase A: boosted wavepacket

Protocol. (i) Compute a rest soliton $S_\star$. (ii) Construct a boosted initial profile by solving the constraint system that enforces a constant aether-frame velocity $\mathbf{v}$ and fixes the momentum density $T^{0i}_{\rm sys}$. (iii) Evolve on a box with absorbing layers and record $E_{\rm sys}(t) = \int_\Sigma T^{00}_{\rm sys}\, d^3x$, the field momentum $\mathbf{P}_{\rm sys}$, and the radiative losses.

Targets. For each $|\mathbf{v}| < c$,

$$E_{\text{sys}}(v) \;=\; \gamma(v)\, M_{\text{eff}} c^2 \left[1 + \mathcal{O}(\eta,\ \text{anisotropy},\ v^4)\right], \qquad \gamma(v) \equiv (1 - v^2/c^2)^{-1/2}, \quad (3.59)$$

and $\mathbf{P}_{\text{sys}}(v) = \gamma M_{\text{eff}} \mathbf{v}\,[1 + \mathcal{O}(\cdots)]$. A *dispersion pass* requires a two-parameter fit (M_{eff}, c) to (3.59) with relative residual $\leq 10^{-3}$ for $v/c \in \{0.2, 0.4, 0.6, 0.8\}$, and negligible spurious $\mathcal{O}(v^3)$ anisotropic terms after angle averaging.

3.6.4 Numerical showcase B: head-on soliton collision

Protocol. Initialize two identical solitons at $\pm z_0$ with velocities $\pm v$. Evolve until well after separation (or annihilation), tracking:

$$E_{\text{tot}}(t) = E_{\text{sys}}(t) + E_{\text{rad}}(t) + E_{\text{bath}}(t), \quad \Delta E_{\text{num}} \equiv \frac{\max_t |E_{\text{tot}}(t) - E_{\text{tot}}(0)|}{E_{\text{tot}}(0)}. \quad (3.60)$$

Extract outgoing rest masses $M_{\text{out}}^{(i)}$, kinetic energies, and a lumped inelasticity $K_{\text{loss}} \equiv E_{\text{rad}}^{\text{out}} + E_{\text{bath}}^{\text{out}}$.

Targets and gates. (i) $\Delta E_{\text{num}} \leq 5 \times 10^{-4}$ (energy closure); (ii) momentum balance within 10^{-3} (accounting for radiative recoil); (iii) absence of pre-acceleration in the approach phase (monotone relation between force and acceleration up to GKSL noise); (iv) recovered outgoing masses consistent with rest calibration to 10^{-3}.

3.6.5 Acceptance metrics and reporting standard

We recommend the following summary table in all calibration runs:

Test	Measured	Target relation	Pass threshold
Compton (rest)	ω_0	$M_{\text{eff}} = \hbar\omega_0/c^2$	$\lvert\delta M\rvert/M \leq 10^{-3}$
Annihilation	$E_{\text{rad}}, E_{\text{bath}}$	$E_{\text{rad}} + E_{\text{bath}} = 2M_{\text{eff}}c^2 - \lvert E_{\text{bind}}^{\text{in}}\rvert$	rel. err. $\leq 10^{-3}$
Binding (static)	$E[S_{(N)}]$	$\Delta m = \lvert E_{\text{bind}}\rvert/c^2$	virial + closure $\leq 1\%$
Boosted packet	$E(v), P(v)$	$E = \gamma Mc^2,\ P = \gamma Mv$	fit residual $\leq 10^{-3}$
Collision ledger	$E_{\text{sys}}, E_{\text{rad}}, E_{\text{bath}}$	Eq. (3.60)	$\Delta E_{\text{num}} \leq 5 \times 10^{-4}$

Each run must state $(M_{\text{eff}}, c, \hbar)$ calibrations, solver tolerances, domain size, boundary treatment, and gate checks (no ghosts, hyperbolicity, $\mathcal{Q}_{\text{diss}} \geq 0$, universal c in IR).

Summary. The ladder above ties together *rest* and *boosted* diagnostics, static *binding* and dynamical *annihilation/collision* calorimetry, and enforces mass–energy consistency across independent probes. Passing these tests within the stated tolerances constitutes an end-to-end validation of $E = M_{\text{eff}}c^2$ in the spectral framework.

3.7 Gates and Scope of Validity

Aim. We state precise *conditions of validity* under which the spectral dispersion reduces to the Einstein mass–energy relation

$$E^2 \;=\; (pc)^2 \;+\; (M_{\mathrm{eff}}c^2)^2, \qquad p^2 \equiv g^{ij}_{\mathrm{eff}}p_i p_j, \tag{3.61}$$

and quantify controlled corrections from anisotropy, higher gradients, and dissipation. The conditions are phrased as *consistency gates*.

Structural & spectral gates (S)

1. **Positive time kinetics (no ghosts).** Let K be the time-kinetic block of the quadratic action (Vol. III, Sec. 1.3). Require

$$\mathsf{K} \succ 0, \qquad \lambda_{\min}(\mathsf{K}) \geq \kappa_{\min} > 0, \tag{3.62}$$

   so that canonical normalization exists and the symplectic form is nondegenerate. This ensures a positive rest energy $M_{\mathrm{eff}}c^2$.

2. **Controlled higher gradients.** Let the principal spatial symbol admit an expansion $\omega^2 = c^2(\hat{\mathbf{k}})k^2 + \omega_0^2 + \alpha_4(\hat{\mathbf{k}})k^4 + \cdots$. Define the crossover scale $k_*^{-2} \equiv \max_{\hat{\mathbf{k}}} |\alpha_4(\hat{\mathbf{k}})|/c^2$. Impose

$$k \ll k_* \quad \Longrightarrow \quad \left| \frac{\alpha_4(\hat{\mathbf{k}})k^4}{c^2k^2 + \omega_0^2} \right| \ll 1, \tag{3.63}$$

   so that quartic and higher corrections are negligible in (3.61).

Geometric & hyperbolicity gates (G)

1. **Causal cone and subluminality.** The principal symbol defines a front speed $v_{\mathrm{front}}(\hat{\mathbf{k}}) = \lim_{k\to\infty} \omega/k$. Require

$$0 < v_{\mathrm{front}}(\hat{\mathbf{k}}) \leq c_{\mathrm{eff}} \equiv 1 \quad \text{(units of the emergent metric)}, \tag{3.64}$$

   and hyperbolicity of the equations of motion in the aether frame (u^μ timelike w.r.t. g_{eff}).

2. **Universal IR speed.** Under RG coarse–graining with small background dissipation ($\eta \downarrow 0^+$), sectoral wave speeds collapse to a common value

$$c^2(\hat{\mathbf{k}}) \;=\; c^2\Big[1 + \epsilon_{\mathrm{aniso}}\, P_2(\cos\theta) + \cdots\Big], \qquad |\epsilon_{\mathrm{aniso}}| \;\ll\; 1, \tag{3.65}$$

   uniformly over the momentum range used in the kinematics test. Angle averaging yields $\langle c^2(\hat{\mathbf{k}})\rangle_\Omega = c^2$.

Topological & quantization gates (TQ)

1. **Finite energy & well–defined rest mass.** For localized (possibly topological) configurations, the energy functional $E[S]$ (Vol. I, Ch. 3) must satisfy

$$E[S_\star] = \int d^3x\, \mathcal{H}[S_\star] < \infty, \quad \delta E|_{S_\star} = 0, \tag{3.66}$$

   with stability under small perturbations (no negative modes in the Hessian). Then $M_{\rm eff} = E[S_\star]/c^2$ is well defined.

2. **Quantization scale and canonical algebra.** The symplectic two–form Ω (Sec. 1.3) induces a positive inner product and a quantization scale $\hbar$ such that the small–k dispersion matches $\omega_0 = M_{\rm eff}c^2/\hbar$.

Dissipative & analyticity gates (D)

1. **Weakly open regime.** Let the GKSL rates γ_a define a total decoherence rate $\Gamma_{\rm tot} \equiv \sum_a \gamma_a \|L_a\|^2$. Require

$$\frac{\Gamma_{\rm tot}}{\omega} \ll 1 \quad \text{for the modes used to infer } (E,p), \tag{3.67}$$

   so that $\omega \to \omega - i\Gamma_{\rm tot}/2$ produces only *narrow* linewidths and the kinematic relation (3.61) holds for the *real* part.

2. **Forward–limit positivity.** For the sector describing boosted solitons, the forward elastic amplitude $\mathcal{A}(s, t=0)$ satisfies the usual positivity bounds, ensuring that higher–derivative operators do not violate causality/analyticity and keeping the strong–coupling scale $\Lambda_{\rm sc}$ above the process energy:

$$E_{\rm proc} \ll \Lambda_{\rm sc} \quad \Rightarrow \quad \text{EFT under control.} \tag{3.68}$$

Summary: validity band and controlled corrections

Under the gates (3.62)–(3.68), the dispersion reduces to (3.61) with corrections

$$\frac{E^2 - \left(p^2c^2 + M_{\rm eff}^2c^4\right)}{M_{\rm eff}^2c^4} = \underbrace{\mathcal{O}\!\left(\epsilon_{\rm aniso}\,\frac{p^2}{M_{\rm eff}^2c^2}\right)}_{\text{angle dependence}} + \underbrace{\mathcal{O}\!\left(\frac{p^4}{M_{\rm eff}^2c^2}\,\frac{1}{(k_*c)^2}\right)}_{\text{higher gradients}} + \underbrace{\mathcal{O}\!\left(\frac{\Gamma_{\rm tot}}{\omega}\right)}_{\text{weak openness}} + \cdots . \tag{3.69}$$

The nonrelativistic limit follows uniformly as $E = M_{\rm eff}c^2 + p^2/(2M_{\rm eff}) + \mathcal{O}(p^4)$, while the ultrarelativistic limit $E \simeq pc$ is reached for $p \gg M_{\rm eff}c$ provided $p \ll \hbar k_*$ and $\Gamma_{\rm tot} \ll \omega$.

Operational note. All numerical and experimental tests in Sec. 3.6 must report $\epsilon_{\rm aniso}$, k/k_*, and $\Gamma_{\rm tot}/\omega$ to certify that they lie inside the *validity band* where (3.61) is guaranteed by the gates.

Part III

Spectral Recovery of Classical and Quantum Physics

Chapter 4

Precision Relativity and Classical Tests

4.1 Setup: Emergent Geodesics, Local Frames, and Effective Lorentz Kinematics

We start from the emergent Lorentzian geometry of Volume I: a coframe $e^a{}_\mu[\mathcal{S}]$ and effective metric $g^{\rm eff}_{\mu\nu} = e^a{}_\mu\,\eta_{ab}\,e^b{}_\nu$, constructed from the spectral state $\mathcal{S} = \{\Phi, \rho, \eta, \chi^\mu, T\}$. Timelike aether frame u^μ defines the 3+1 split and the local notion of speed. Worldlines of freely-falling compact probes obey

$$u^\nu \nabla^{\rm eff}_\nu u^\mu \;=\; 0, \qquad u^\mu u^\nu g^{\rm eff}_{\mu\nu} = -1, \tag{4.1}$$

with $\nabla^{\rm eff}$ the Levi–Civita connection of $g^{\rm eff}$. In a local inertial patch $\mathcal{U}$ where $g^{\rm eff}_{\mu\nu} = \eta_{\mu\nu} + \mathcal{O}(x^2)$ and torsionless connection vanishes at the origin, the kinematics reduces to an *effective Lorentz group* with speed $c \equiv c_{\rm eff} = 1$ in units where the IR metric light-cone sets the causal cone.

Small anisotropies inherited from the internal director $\hat{\chi}$ enter at higher orders through the principal symbol of fluctuations:

$$\omega^2 \;=\; c_\parallel^2\,k_\parallel^2 + c_\perp^2\,k_\perp^2 + m_{\rm eff}^2 + \cdots, \qquad 0 < c_\perp^2 \le c_\parallel^2 \le 1, \tag{4.2}$$

but the *Lorentz transformations* between comoving observers are governed by $g^{\rm eff}$: for a relative 3–velocity $\mathbf{v}$ (measured in the u^μ frame),

$$\gamma \;=\; \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}, \qquad \Lambda^\mu{}_\nu(\mathbf{v}) \text{ is defined by } \Lambda^\top g^{\rm eff}\Lambda = g^{\rm eff}, \tag{4.3}$$

with anisotropic *corrections to dispersion* captured by (4.2) but *not* by a modification of the causal cone (Consistency Gate D).

4.1.1 Equivalence principle in the emergent medium

Minimal coupling of compact matter solitons to $g^{\rm eff}$ implies universality of free-fall to leading order: the inertial mass $m_{\rm in}$ equals the gravitational charge $m_{\rm grav}$ extracted from the source term in the effective Einstein equations (Vol. I, Ch. 3), $m_{\rm in} = m_{\rm grav}[\,\mathcal{S}\,] + \mathcal{O}(\epsilon^2_{\rm aniso})$, with $\epsilon_{\rm aniso} \sim |c_\parallel - c_\perp|$. Violations appear only at second order in anisotropic/stiffness contrasts and are gate-suppressed by $|c_\parallel - c_\perp| \ll 1$ and positivity of the time-kinetic blocks (Gate S).

4.2 Post-Newtonian Expansion and Mapping to PPN

Consider a weak, slowly-varying field in the aether rest frame. Write $g_{00}^{\text{eff}} = -(1 + 2U + 2\beta U^2 + \cdots)$, $g_{0i}^{\text{eff}} = -(1+\gamma)V_i + \cdots$, $g_{ij}^{\text{eff}} = (1 - 2\gamma U)\delta_{ij} + \cdots$, with U the Newtonian potential and V_i gravitomagnetic potential. Matching the emergent field equations to the *Parameterized Post-Newtonian* (PPN) metric yields

$$\begin{aligned} \gamma_{\text{PPN}} &= 1 + \delta\gamma(\zeta_{\text{spec}}), & \beta_{\text{PPN}} &= 1 + \delta\beta(\zeta_{\text{spec}}), \\ \alpha_1 &= \alpha_1(\zeta_{\text{spec}}), \quad \alpha_2 = \alpha_2(\zeta_{\text{spec}}), \quad \xi = \xi(\zeta_{\text{spec}}), \ \ldots \end{aligned} \tag{4.4}$$

where ζ_{spec} abbreviates the spectral couplings controlling the curvature sector and stiffness anisotropy (e.g. $M_{\text{eff}}(\rho)$, $Z_\perp$, $\Delta Z \equiv Z_\parallel - Z_\perp$, and small non-minimal mixing terms). Gate G (covariance in the emergent geometry) sets $\alpha_{1,2}, \xi \to 0$ in the IR; Gate D (causality) constrains $\delta\gamma, \delta\beta$ via positivity/dispersion bounds (Appendix **??**).

Observable consequences. To leading PN order one recovers the standard classical tests with small spectral corrections:

$$\text{Shapiro delay:} \quad \Delta t = (1 + \gamma_{\text{PPN}})\frac{GM}{c^3}\,\ln\frac{r_E + r_R + R}{r_E + r_R - R}\,\left[1 + \mathcal{O}(\epsilon_{\text{aniso}})\right], \tag{4.5}$$

$$\text{Perihelion advance:} \quad \Delta\varpi = \frac{6\pi GM}{a(1-e^2)c^2}\,\left[1 + \delta_\varpi(\delta\beta, \delta\gamma, \epsilon_{\text{aniso}})\right], \tag{4.6}$$

$$\text{Lense–Thirring:} \quad \boldsymbol{\Omega}_{\text{LT}} = \frac{2G\mathbf{J}}{c^2 r^3}\left[1 + \delta_{\text{LT}}(\alpha_{1,2}, \epsilon_{\text{aniso}})\right], \tag{4.7}$$

with a, e the orbital elements and $\mathbf{J}$ the source angular momentum. Current bounds force $|\delta\gamma|, |\delta\beta|, |\alpha_{1,2}|, |\epsilon_{\text{aniso}}|$ to be $\ll 10^{-5}$–10^{-3} depending on the channel; we translate these to allowed domains in $(Z_\perp, \Delta Z, M_{\text{eff}})$ in Sec. 4.4.

4.3 Gravitational-Wave Speed and Ringdown

Tensor perturbations of g^{eff} obey

$$S_T^{(2)} = \frac{M_{\text{eff}}^2}{8}\int d^4x\, a^3\left[\dot{h}_{ij}^2 - c_T^2\, a^{-2}(\partial_k h_{ij})^2\right], \qquad c_T^2 = 1 + \delta c_T(\zeta_{\text{spec}}), \tag{4.8}$$

hence GW front speed $v_{\text{GW}} = c_T$. Multi-messenger constraints imply $|\delta c_T| \lesssim \mathcal{O}(10^{-15})$ today, which maps to tight inequalities on curvature mixing and anisotropic stiffness. In strong-field ringdown, the quasi-normal spectrum of a Kerr-like background is shifted via the effective potential $V_{\text{eff}}\left[c_T, M_{\text{eff}}, \ldots\right]$:

$$\frac{\delta\omega_{n\ell m}}{\omega_{n\ell m}} \;=\; A_{n\ell m}\,\delta c_T + B_{n\ell m}\,\frac{\delta M_{\text{eff}}}{M_{\text{eff}}} \;+\; \mathcal{O}(\epsilon_{\text{aniso}}^2), \tag{4.9}$$

with (A, B) calculable from the Teukolsky-like radial problem in the emergent background. Gate D demands $|c_T - 1| \ll 1$; Gate S enforces positive kinetic energy for tensors; both together rule out large deviations in (4.8).

Observable / Test	**PPN / Speed Param.**	**Spectral (linearized) map**
Shapiro time delay	$\delta\gamma \equiv \gamma_{\rm PPN} - 1$	$\delta\gamma \simeq (M_{\rm PPN})_\gamma \cdot (\Delta Z,\ Z_\perp/M_{\rm eff}^2,\ \zeta_{\rm mix})$
Perihelion advance	$\delta\beta \equiv \beta_{\rm PPN} - 1$	$\delta\beta \simeq (M_{\rm PPN})_\beta \cdot (\Delta Z,\ Z_\perp/M_{\rm eff}^2,\ \zeta_{\rm mix})$
Preferred-frame effects	$\alpha_1,\ \alpha_2$	$\alpha_i \simeq (M_{\rm PPN})_{\alpha_i} \cdot (\zeta_{\rm mix},\ \partial_\rho \ln M_{\rm eff})$
Frame-dragging (Lense–Thirring)	$\Omega_{\rm LT}$ via $\alpha_{1,2}$	$\delta\Omega_{\rm LT} \sim A_1\alpha_1 + A_2\alpha_2$
GW propagation speed	$\delta c_T \equiv c_T - 1$	$\delta c_T \simeq (M_T) \cdot \zeta_{\rm curv}$
Dispersion anisotropy	$\epsilon_{\rm aniso} \equiv \lvert c_\parallel - c_\perp \rvert$	$\epsilon_{\rm aniso} \simeq B_1\, \Delta Z/Z_\perp + B_2\, \zeta_{\rm mix}$

Table 4.1: Mapping of precision-relativity observables to spectral parameters. The matrices/vectors $(M_{\rm PPN})_{\gamma,\beta,\alpha_i}$ and M_T, and coefficients $A_{1,2}, B_{1,2}$ are calculable from the curvature/stiffness sector specified in Vol. I and the weak-field matching in Secs. 4.2–4.3. Admissible regions are those satisfying the gates in Eq. (4.10).

4.4 Consistency Gates and Mapping to Spectral Parameters

We collect the *precision* constraints as inequalities on the spectral couplings:

$$\begin{aligned}
\textbf{(S) No ghosts:}\quad & M_{\rm eff}^2 > 0, \quad \text{time-kinetic blocks } \succ 0,\\
\textbf{(G) Covariant IR:}\quad & \alpha_{1,2}, \xi \to 0, \quad \nabla^{\rm eff} g^{\rm eff} = 0,\\
\textbf{(D) Causality/hyperbolicity:}\quad & 0 < c_T^2 \le 1,\ 0 < c_\perp^2 \le c_\parallel^2 \le 1,\\
\textbf{(PPN bounds):}\quad & |\delta\gamma|, |\delta\beta| \lesssim 10^{-5},\ |\alpha_{1,2}| \lesssim 10^{-5},\ |\epsilon_{\rm aniso}| \lesssim 10^{-3},\\
\textbf{(GW speed):}\quad & |\delta c_T| \lesssim 10^{-15} \text{ (today)}.
\end{aligned} \tag{4.10}$$

A minimal linear map suffices for forecasting:

$$\begin{pmatrix} \delta\gamma \\ \delta\beta \\ \alpha_1 \\ \alpha_2 \\ \delta c_T \end{pmatrix} = \mathbf{M}_{\rm PPN} \begin{pmatrix} \Delta Z \\ Z_\perp/M_{\rm eff}^2 \\ \zeta_{\rm mix} \\ \cdots \end{pmatrix} + \mathcal{O}(\zeta^2), \qquad \|\mathbf{M}_{\rm PPN}\| \text{ fixed by Vol. I curvature sector.} \tag{4.11}$$

The allowed region in $(\Delta Z, Z_\perp, \zeta_{\rm mix})$ is the preimage of the rectangular box defined by (4.10). This *safety domain* will be overlaid on all subsequent fits.

Outcome. Within the gates (4.10), the spectral medium reproduces: (i) effective Lorentz transformations tied to $g^{\rm eff}$, (ii) the Einstein equivalence principle at leading order, (iii) PPN phenomenology with $\gamma, \beta \simeq 1$, (iv) classical tests (Shapiro, perihelion, Lense–Thirring) up to tiny, bounded corrections, and (v) $c_T \simeq 1$ with ringdown shifts below current sensitivity. This *closes* the "equivalence" and "causal cone" questions for the recovery program.

Chapter 5

Emergent Electromagnetism and Energy–Momentum Forms

Aim. We derive an effective $U(1)$ gauge sector as an isometry of the internal spectral fiber, define electric charge as a Noether current, obtain Maxwell equations on the emergent geometry g_{eff} with a state–dependent constitutive tensor, prove the Poynting theorem and the stress–energy tensor in the spectral medium, and quantify birefringence/causality constraints inherited from anisotropy along $\hat{\chi}$.

5.1 U(1) from Fiber Isometries and the Spectral Connection

Fiber isometry. Let $\mathcal{F}$ denote the internal spectral fiber for the phase field Φ (Vol. I, Ch. 2). A one–parameter subgroup of isometries $\varphi_\alpha : \Phi \mapsto e^{i\alpha}\Phi,\ \alpha \in \mathbb{R}/2\pi\mathbb{Z}$ acts on $\mathcal{F}$ with Killing generator K_Φ. Promote $\alpha = \alpha(x)$ to a *local* parameter. To preserve the invariance of the spectral kinetic term, introduce the *spectral gauge connection* A_μ as the K_Φ–component of the emergent connection ω_μ:

$$D_\mu\Phi \;\equiv\; \nabla_\mu\Phi - iq\,A_\mu\,\Phi, \qquad A_\mu \;\equiv\; \omega_\mu(K_\Phi), \quad \Phi(x) \to e^{iq\alpha(x)}\Phi(x), \quad A_\mu \to A_\mu + \partial_\mu\alpha. \tag{5.1}$$

Here q is the spectral charge quantum determined by the normalization of K_Φ.

Field strength and curvature. The $U(1)$ curvature arises from the commutator of covariant derivatives:

$$F_{\mu\nu} \;\equiv\; \partial_\mu A_\nu - \partial_\nu A_\mu \;=\; (\mathrm{d}A)_{\mu\nu}, \qquad [D_\mu, D_\nu]\Phi \;=\; -\,iq\,F_{\mu\nu}\,\Phi + (\text{curvatures orthogonal to } K_\Phi). \tag{5.2}$$

Spectral origin of the photon. Small fluctuations of A_μ around ordered states S_0 are collective excitations of the response field χ^μ projected along K_Φ. Their kinetic term and light cone are inherited from the quadratic action of Vol. I (Ch. 3) and constrained by the gates (no ghosts, hyperbolicity).

5.2 Noether Charge and Effective Maxwell on $g_{\rm eff}$

Action and constitutive response. The most general local, gauge–invariant and parity–even quadratic action on the emergent geometry reads

$$\mathcal{L}_{\rm EM} = -\frac{1}{4}\,\chi^{\mu\nu\rho\sigma}[\mathcal{S}]\,F_{\mu\nu}F_{\rho\sigma} \;+\; \mathcal{L}_{\rm matt}[\Phi,\ldots;D_\mu] \;-\; A_\mu J^\mu_{\rm ext}, \tag{5.3}$$

where $\chi^{\mu\nu\rho\sigma}$ is the *constitutive tensor* built from $(g_{\rm eff}, u^\mu, \hat{\chi}^\mu, \rho, \eta)$ with the symmetries $\chi^{\mu\nu\rho\sigma} = -\chi^{\nu\mu\rho\sigma} = -\chi^{\mu\nu\sigma\rho} = \chi^{\rho\sigma\mu\nu}$.

Maxwell equations on $g_{\rm eff}$. Define $H^{\mu\nu} \equiv \chi^{\mu\nu\rho\sigma}F_{\rho\sigma}$. Variation of (5.3) yields

$$\nabla_\mu H^{\mu\nu} \;=\; J^\nu \equiv J^\nu_{\rm matt} + J^\nu_{\rm ext}, \qquad \nabla_{[\mu}F_{\rho\sigma]} \;=\; 0, \tag{5.4}$$

with covariant derivative ∇ compatible with $g_{\rm eff}$. Gauge invariance implies charge conservation: $\nabla_\nu J^\nu = 0$.

Noether current (spectral charge). For $\mathcal{L}_{\rm matt}[\Phi; D_\mu]$, a global phase $\Phi \to e^{iq\alpha}\Phi$ gives the Noether current

$$J^\mu_{\rm matt} = q\ {\rm Im}(\Phi^* D^\mu \Phi) \;+\; (\text{higher sectors/derivative corrections}), \qquad Q = \int_\Sigma d\Sigma_\mu\, J^\mu. \tag{5.5}$$

5.3 Constitutive Tensor, Fermat Principle, and Birefringence

Medium decomposition. Relative to the aether frame u^μ and spatial projector $h^{\mu\nu} = g^{\mu\nu}_{\rm eff} + u^\mu u^\nu$, decompose $F_{\mu\nu}$ into electric and magnetic fields:

$$E_\mu \equiv F_{\mu\nu}u^\nu, \qquad B_\mu \equiv \tfrac{1}{2}\,\epsilon_{\mu\nu\rho}\,F^{\nu\rho}, \qquad \epsilon_{\mu\nu\rho} \equiv \epsilon_{\mu\nu\rho\sigma}u^\sigma, \tag{5.6}$$

and constitutive relations

$$D^\mu = \varepsilon^\mu{}_\nu\,E^\nu + \gamma^\mu{}_\nu\,B^\nu, \qquad H^\mu = \mu^{-1\,\mu}{}_\nu\,B^\nu + \tilde{\gamma}^\mu{}_\nu\,E^\nu, \tag{5.7}$$

with tensors $(\varepsilon, \mu^{-1}, \gamma, \tilde{\gamma})$ functions of $(\rho, \eta, \hat{\chi})$. Passivity/positivity demand $E_\mu D^\mu + B_\mu H^\mu \geq 0$.

Eikonal (Fermat) limit. Let $A_\mu = a_\mu\, e^{i\theta/\epsilon}$, $\epsilon \to 0$, $k_\mu \equiv \nabla_\mu\theta$. Maxwell equations reduce to an eigenvalue problem for the polarization a_μ:

$$\mathsf{M}^\mu{}_\nu(k)\,a^\nu = 0, \qquad \mathsf{M}^\mu{}_\nu(k) \equiv k_\alpha k_\beta\,\chi^{\alpha\mu\beta}{}_\nu. \tag{5.8}$$

Nontrivial solutions require $\det \mathsf{M}(k) = 0$, yielding the *Fresnel (dispersion) surface*. For isotropic response, this collapses to a single null cone of an *optical metric* $g^{\mu\nu}_{\rm opt}$ (Gordon form)

$$g^{\mu\nu}_{\rm opt}\,k_\mu k_\nu = 0, \qquad g^{\mu\nu}_{\rm opt} = g^{\mu\nu}_{\rm eff} + \left(1 - n^{-2}\right)u^\mu u^\nu, \quad n^2 \equiv \varepsilon\,\mu. \tag{5.9}$$

Anisotropy along $\hat{\chi}$ generically splits the surface into two sheets (birefringence): $g^{\mu\nu}_{\rm opt} \to \{g^{\mu\nu}_{{\rm opt},+},\ g^{\mu\nu}_{{\rm opt},-}\}$ with polarization–dependent indices $n_\pm(\theta)$ and phase/group velocities $c_\pm(\theta)$, $\theta \equiv \angle(\mathbf{k}, \hat{\chi})$.

Consistency gates. Causality demands the optical cones lie *inside* the g_{eff} cone:

$$0 < c_{\pm}^2(\theta) \le 1 \quad \forall\, \theta, \qquad \big|c_+(\theta) - c_-(\theta)\big| \ll 1 \ \text{(birefringence bounds)}. \tag{5.10}$$

5.4 Poynting Theorem and Stress–Energy in the Spectral Medium

Stress–energy tensor. Varying (5.3) with respect to $g_{\text{eff}}^{\mu\nu}$ at fixed $\chi^{\mu\nu\rho\sigma}$ gives

$$T^{\mu\nu}_{\text{EM}} = H^{\mu\alpha}F^{\nu}{}_{\alpha} - \frac{1}{4}\, g^{\mu\nu}_{\text{eff}}\, H^{\alpha\beta}F_{\alpha\beta}, \qquad H^{\mu\nu} \equiv \chi^{\mu\nu\rho\sigma}F_{\rho\sigma}. \tag{5.11}$$

In the aether frame, define energy density and Poynting vector

$$u_{\text{EM}} \equiv \tfrac{1}{2}\Big(E\cdot D + B\cdot H\Big), \qquad S^{\mu} \equiv \epsilon^{\mu}{}_{\nu\rho}\, E^{\nu}H^{\rho}, \tag{5.12}$$

and momentum density $\Pi^{\mu} \equiv S^{\mu}$ (in units $c = 1$), modulo Abraham–Minkowski conventions fixed by χ.

Poynting theorem with dissipation. Contract (5.4) with E_ν and the Bianchi with H_ν; after algebra,

$$\nabla_\mu S^{\mu} \;+\; \partial_t u_{\text{EM}} \;=\; -\,J_\mu E^{\mu} \;-\; \mathcal{Q}_{\text{diss}}, \qquad \mathcal{Q}_{\text{diss}} \equiv \tfrac{1}{2}\,\dot{\chi}^{\mu\nu\rho\sigma}F_{\mu\nu}F_{\rho\sigma} \;\ge 0, \tag{5.13}$$

where the explicit time dependence of χ (through $\eta > 0$ channels) produces positive semi–definite dissipation $\mathcal{Q}_{\text{diss}}$, consistent with entropy production (Vol. I gates). Equation (5.13) certifies energy–momentum bookkeeping in the medium: work done on charges plus heating equals EM energy loss.

5.5 Consistency Gates and Observational Touchstones

Gates.

1. **No ghosts:** the quadratic form $E\cdot D + B\cdot H > 0$ for all (E, B).
2. **Hyperbolicity/causality:** the Fresnel surface defines forward cones within that of g_{eff}; front velocities ≤ 1.
3. **Passivity/positivity:** $\mathcal{Q}_{\text{diss}} \ge 0$ for $\dot{\chi}$ driven by $\eta > 0$.
4. **Weak birefringence:** $|c_+(\theta) - c_-(\theta)| \ll 1$ in the appropriate observational band.

Calibrations (touchstones).

- *Quasi–static limit:* $\nabla\cdot D = \rho_{\text{free}}$, $\nabla\times H = J_{\text{free}}$ with ε, μ evaluated on S_0 reproduce Coulomb/Ampère at lab scales.
- *Ray optics:* Fermat paths follow null geodesics of g_{opt}; lensing matches the Gordon metric built from (n, u^{μ}).
- *Polarization transport:* Berry/Faraday rotation computed from χ agrees with small anisotropy bounds; any residual cosmic birefringence maps to $(\varepsilon, \mu, \gamma)$ anisotropies.

Remark (bridge to gauge sector). The construction generalizes: isometries of the internal fiber $\mathcal{F}$ beyond the $U(1)$ phase yield non–Abelian connections as components of ω_μ along Killing directions; their kinetic terms descend from the curvature sector of Vol. I, subject to the same S/G gates.

Chapter 6

Scattering Theory and the S–Matrix on a Spectral Background

Aim. Build a micro–consistent scattering framework on the emergent geometry g_{eff} with small asymptotic dissipation $\eta\downarrow 0$: (i) define in/out asymptotics and wave operators, (ii) derive LSZ with anisotropic dispersion, (iii) state the Källén–Lehmann representation and microcausality, (iv) prove the (generalized) Optical Theorem, (v) impose forward–limit positivity bounds, and (vi) map spectral couplings to cross sections.

6.1 Asymptotic Regions, Wave Operators, and the S–Map

Asymptotia. Assume a stationary ordered background $S_\infty = \{\rho_\infty, \eta_\infty \geq 0, \hat{\chi}^\mu_\infty, T_\infty\}$ such that

$$\lim_{t\to\pm\infty} \partial_t S = 0, \qquad \eta_\infty \to 0^+, \qquad g_{\text{eff}} \to g^{(\infty)}_{\text{eff}}, \tag{6.1}$$

so free dynamics is generated by H_0 on $g^{(\infty)}_{\text{eff}}$ with dispersion $\omega_\alpha(\mathbf{k},\theta)$ for each normal mode α (Volume II). Let $U(t,s)$ and $U_0(t,s)$ be full and free propagators.

Møller operators and scattering map. With adiabatic switching $H \mapsto H\, e^{-\epsilon|t|}$,

$$\Omega_\pm \;\equiv\; \mathrm{s}\!-\!\lim_{t\to\pm\infty} U(0,t)\, U_0(t,0), \qquad \mathsf{S} \equiv \Omega_+^\dagger \Omega_-. \tag{6.2}$$

If $\eta_\infty = 0$ one obtains *unitary* S on the Fock space built in Sec. 1.3. For small but nonzero dissipation in the asymptotic zone, scattering is a completely positive, trace–preserving (CPTP) map on density matrices

$$\mathcal{S}(\rho) \;=\; \sum_n K_n\, \rho\, K_n^\dagger, \qquad \sum_n K_n^\dagger K_n = \mathbb{1}, \tag{6.3}$$

reducing to $\mathsf{S}\rho\,\mathsf{S}^\dagger$ as $\eta_\infty\downarrow 0$.

6.2 LSZ Reduction with Anisotropic Dispersion

Consider a scalar mode φ (a canonically–normalized combination of spectral perturbations). Define the on–shell poles by

$$k_0 = \omega(\mathbf{k},\theta), \qquad \omega^2(\mathbf{k},\theta) = c^2(\theta)\, k^2 + m^2 c^4 \;+\; \mathcal{O}(k^4/\Lambda^2), \qquad \theta \equiv \angle(\mathbf{k}, \hat{\chi}_\infty). \tag{6.4}$$

Let $Z(\theta)$ be the residue (wavefunction renormalization) of the time–ordered 2–point function Δ_F at the pole. Then the LSZ formula for $n\to m$ scattering reads

$$\langle \mathbf{p}_1,\dots,\mathbf{p}_m \text{ out} \,|\, \mathbf{k}_1,\dots,\mathbf{k}_n \text{ in}\rangle \;=\; i^{n+m}\prod_{i=1}^{n+m}\Big[Z^{-1/2}(\theta_i)\int d^4x_i\, e^{\pm ik_i\cdot x_i}\big(\Box_{g_{\text{eff}}}+m^2\big)\Big] \times\Big\langle \mathrm{T}\,\varphi(x_1)\cdots\varphi(x_{n+m})\Big\rangle\Big|_{\{k_0=\pm\omega(\mathbf{k},\theta)\}}, \tag{6.5}$$

with upper (lower) sign for outgoing (incoming) legs, and $\Box_{g_{\text{eff}}}$ computed with $g^{(\infty)}_{\text{eff}}$. The phase–space measure adapts to $\omega(\mathbf{k},\theta)$:

$$d\Pi(\mathbf{k}) \;=\; \frac{d^3\mathbf{k}}{(2\pi)^3\,2\,\omega(\mathbf{k},\theta)}. \tag{6.6}$$

When $c(\theta)\to c$, the usual isotropic LSZ is recovered.

6.3 Källén–Lehmann Representation and Microcausality

For any (Heisenberg) scalar operator φ, the 2–point function admits

$$\Delta_F(k;\hat{\mathbf{k}}) = \int_0^\infty d\mu^2\;\frac{\rho(\mu^2;\hat{\mathbf{k}})}{k_0^2-\omega_\mu^2(\mathbf{k},\theta)+i\epsilon}, \qquad \rho(\mu^2;\hat{\mathbf{k}})\ge 0, \quad \int d\mu^2\,\rho = Z(\hat{\mathbf{k}})\le 1, \tag{6.7}$$

where ω_μ is the dispersion with mass parameter μ, and $\hat{\mathbf{k}}\equiv\mathbf{k}/|\mathbf{k}|$. *Microcausality* on $g^{(\infty)}_{\text{eff}}$ implies the commutator vanishes for spacelike separations w.r.t. that cone; this ensures analyticity of amplitudes in the upper/lower half–planes and underlies dispersion relations.

6.4 Optical Theorem: Unitarity and its CP Extension

With $\eta_\infty=0$, $\mathsf{S}^\dagger\mathsf{S}=1$ gives the optical theorem for the forward amplitude $\mathcal{M}_{ii}(s,t=0)$:

$$2\,\mathrm{Im}\,\mathcal{M}_{ii}(s,0) \;=\; \sum_f\int d\Pi_f\,|\mathcal{M}_{fi}(s,t)|^2 \;\equiv\; s\,\sigma_{\text{tot}}(s), \tag{6.8}$$

with s computed from the emergent metric ($g^{(\infty)}_{\text{eff}}$) and the dispersion (6.4).

If asymptotically there remains weak dissipation (CPTP map (6.3)), the theorem generalizes to

$$2\,\mathrm{Im}\,\mathcal{M}_{ii}(s,0) \;=\; \sum_f\int d\Pi_f\,|\mathcal{M}_{fi}|^2 \;+\; \mathcal{A}_{\text{abs}}(s), \qquad \mathcal{A}_{\text{abs}}(s) \;=\; \sum_n\langle i|K_n^\dagger K_n|i\rangle \;\ge 0, \tag{6.9}$$

interpreted as *absorption* into unresolved channels governed by the GKSL rates (Sec. 1.4). Thus unitarity $\Rightarrow$ positivity of total rates remains intact.

6.5 Forward Dispersion Relations and Positivity Bounds

Assuming (i) microcausality on $g_{\rm eff}^{(\infty)}$, (ii) polynomial boundedness of amplitudes, and (iii) a mass gap (or suitable IR regulator), the forward amplitude admits subtracted dispersion relations. For a scalar $2\to2$ amplitude $\mathcal{M}(s,t)$,

$$\frac{\partial^2}{\partial s^2}\,\mathcal{M}(s,0)\Big|_{s=0} = \frac{2}{\pi}\int_{s_{\rm th}}^{\infty}\frac{ds'}{s'^3}\,\mathrm{Im}\,\mathcal{M}(s',0)\;>\;0. \tag{6.10}$$

Matching $\mathcal{M}$ to the low–energy EFT with higher–derivative operators (e.g. $(\partial\varphi)^4$, F^4) implies *positivity constraints* on the corresponding Wilson coefficients. In anisotropic media, one obtains *directional* bounds by fixing $\hat{\mathbf{k}}$ (or averaging over θ), yielding inequalities along and across $\hat{\chi}_\infty$ that guarantee hyperbolicity and subluminal front velocities.

Growth bounds. With a mass gap $m_{\rm gap}$ and causal cone of $g_{\rm eff}^{(\infty)}$, total cross sections satisfy a Froissart–Martin–type bound

$$\sigma_{\rm tot}(s)\;\lesssim\;\frac{\pi}{m_{\rm gap}^2}\,\log^2\!\Big(\frac{s}{s_0}\Big), \tag{6.11}$$

modulo directional constants set by the optical metric (Sec. 5.3).

6.6 From Spectral Couplings to Cross Sections

Toy 2→2 amplitude. Let φ interact via $\mathcal{L}_{\rm int} = -\frac{\lambda}{4!}\varphi^4 - g^2\,\varphi^2\Xi$, where Ξ is a mediator (e.g. projected χ–fluctuation). The tree–level amplitude

$$\mathcal{M}(s,t,u;\theta_i) = -\lambda\;-\;g^2\!\left(\frac{1}{t-m_\Xi^2}+\frac{1}{u-m_\Xi^2}\right), \tag{6.12}$$

evaluated with kinematics on $g_{\rm eff}^{(\infty)}$, yields the differential cross section

$$\frac{d\sigma}{d\Omega}\;=\;\frac{1}{64\pi^2\,s}\,\frac{\omega(\mathbf{k}_f,\theta_f)}{\omega(\mathbf{k}_i,\theta_i)}\,|\mathcal{M}|^2, \tag{6.13}$$

where the Jacobian (ratio of energies) depends on the anisotropic dispersion. Matching measured $\sigma_{\rm tot}$, $d\sigma/d\Omega$ fixes (λ, g, m_Ξ) as *functions of* spectral couplings in Vol. I–II (entries of $\mathsf{K}, \mathsf{Z}^{ij}, \mathsf{M}^2$).

Partial waves. Define $a_\ell(\sqrt{s};\hat{\chi})$ through the appropriate spherical–harmonic basis on the optical metric. Unitarity implies $|a_\ell| \le 1$ (or $\mathrm{Im}\,a_\ell \ge |a_\ell|^2$), direction–by–direction; violations signal breakdown of the EFT (strong coupling) before the scale $\Lambda_{\rm sc}$.

6.7 Gates and Scope

- **(G)** No ghosts: positive spectral density $\rho \ge 0$; residues $Z(\hat{\mathbf{k}}) \in (0,1]$.
- **(S)** Hyperbolicity/causality: front velocities of all sectors ≤ 1 w.r.t. $g_{\rm eff}^{(\infty)}$; microcausality of commutators.

- **(TQ)** Unitarity/CP: optical theorem (6.8) or CP–extended form (6.9); positivity of total rates.
- **(D)** Positivity bounds: forward–limit inequalities (6.10) satisfied (directionally or averaged); growth bounded as in (6.11).

Outcome. The spectral framework admits a fully consistent scattering theory that reduces to standard collider phenomenology in the isotropic, weak–dissipation limit, while providing *extra* angle–dependent and medium–dependent handles to confront data and bound spectral couplings.

Chapter 7

Quantum Structure: Path Integrals, Coherent States, and Born as an Effective Limit

Aim. Consolidate the "Damascus Interpretation" by (i) completing the construction of the Hilbert space from the quadratic spectral action, (ii) formulating the spectral path integral on the emergent geometry g_{eff}, (iii) deriving coherent (near–classical) sectors, and (iv) showing how the Born rule arises as a stabilization/decoherence limit for $\eta > 0$, with quantitative bounds on deviations.

7.1 Hilbert Space Completion from the Quadratic Spectral Action

Normal modes and complex structure. Starting from the quadratic Lagrangian (cf. Vol. III, Sec. 1.3)

$$\mathcal{L}^{(2)} = \tfrac{1}{2}\,\delta\mathbf{S}^{\top}\Big[\mathsf{K}\,\partial_t^2 + \mathsf{C}\,\partial_t + \mathsf{Z}^{ij}\nabla_i\nabla_j + \mathsf{M}^2\Big]\,\delta\mathbf{S}, \tag{7.1}$$

diagonalization on the ordered background S_0 yields decoupled normal modes $\{\varphi_\alpha\}$ with anisotropic dispersion $\omega_\alpha^2(\mathbf{k},\theta) = c_\alpha^2(\theta)k^2 + m_\alpha^2 c^4 + \cdots$. Define the phase space $\Gamma = \{(\varphi_\alpha, \pi_\alpha)\}$ with symplectic form

$$\Omega\Big((\varphi_1,\pi_1),(\varphi_2,\pi_2)\Big) = \sum_\alpha \int_\Sigma d^3x\,\Big(\varphi_{1,\alpha}\,\pi_{2,\alpha} - \varphi_{2,\alpha}\,\pi_{1,\alpha}\Big). \tag{7.2}$$

A complex structure $\mathsf{J}:\Gamma\to\Gamma$ (compatible with Ω and positive energy) induces the inner product

$$\langle X, Y\rangle \;=\; \tfrac{1}{2}\,\Omega(X,\mathsf{J}Y)\,, \qquad X,Y\in\Gamma, \tag{7.3}$$

whose one–particle completion $\mathcal{H}_1$ and symmetric Fock space $\mathcal{F} = \mathcal{S}(\oplus^n\mathcal{H}_1)$ define the *free* Hilbert sector around S_0.

Creation/annihilation operators. For each mode,

$$\hat\varphi_\alpha(x) = \int \frac{d^3\mathbf{k}}{(2\pi)^3\sqrt{2\omega_\alpha(\mathbf{k},\theta)}}\,\Big(\hat a_{\alpha,\mathbf{k}}\,u_{\alpha,\mathbf{k}}(x) + \hat a_{\alpha,\mathbf{k}}^\dagger\,u_{\alpha,\mathbf{k}}^*(x)\Big)\,, \quad \Big[\hat a_{\alpha,\mathbf{k}},\hat a_{\beta,\mathbf{q}}^\dagger\Big] = (2\pi)^3\delta_{\alpha\beta}\delta^{(3)}(\mathbf{k}-\mathbf{q}). \tag{7.4}$$

Here $u_{\alpha,\mathbf{k}}$ are positive–frequency solutions on $g_{\rm eff}[S_0]$. Solitonic sectors are handled by coherent/displaced representations (Sec. 7.3).

7.2 Spectral Path Integral on $g_{\rm eff}$

Closed–time–path (CTP) generating functional. Quantum dynamics and open–system effects are encoded by the CTP functional on $g_{\rm eff}$:

$$\mathcal{Z}[J_+,J_-]=\int\mathcal{D}\varphi_+\,\mathcal{D}\varphi_-\;\exp\Big\{i\big(S^{(2)}[\varphi_+]-S^{(2)}[\varphi_-]\big)+i\int d^4x\sqrt{-g_{\rm eff}}\,(J_+\varphi_+-J_-\varphi_-)\Big\}\,\mathcal{F}[\varphi_+,\varphi_-], \tag{7.5}$$

where $\mathcal{F}$ is the *influence functional* obtained by integrating out instrument/environmental degrees of freedom (dense/viscous sectors ρ,η and pointer fields; cf. Sec. 1.4). For weak, stationary couplings:

$$\ln\mathcal{F}=-\frac{1}{2}\int d^4x\,d^4y\,\sqrt{-g}\,\Big[\Delta(x)\,\mathsf{D}(x,y)\,\Sigma(y)-\tfrac{i}{2}\,\Delta(x)\,\mathsf{N}(x,y)\,\Delta(y)\Big]+\cdots, \tag{7.6}$$

with $\Sigma\equiv\frac{1}{2}(\varphi_++\varphi_-)$, $\Delta\equiv\varphi_+-\varphi_-$, and retarded kernel D (dissipation) and noise kernel $\mathsf{N}\geq 0$ obeying a fluctuation–dissipation relation governed by η and the spectral density of the apparatus.

From CTP to GKSL. A Markovian, short–memory limit of (7.6) (local in the aether frame u^μ) yields a master equation of the Lindblad/GKSL form for the reduced state ρ (Vol. III, Sec. 1.4):

$$\dot\rho=-i[H_{\rm eff},\rho]-\sum_a\gamma_a\int d^3x\,\Big(L_a\rho L_a^\dagger-\tfrac{1}{2}\{L_a^\dagger L_a,\rho\}\Big),\qquad \gamma_a\sim\mathsf{N}_a,\quad L_a\sim\text{local functionals of }\varphi. \tag{7.7}$$

Causality is ensured since kernels vanish outside the $g_{\rm eff}$ light cone, and complete positivity follows from $\mathsf{N}\geq 0$.

7.3 Coherent States, Classicality, and Wigner Transport

Displacement and minimal uncertainty. For a given mode, $|\alpha\rangle=\exp\big(\alpha\hat a^\dagger-\alpha^*\hat a\big)|0\rangle$ satisfies $\hat a|\alpha\rangle=\alpha|\alpha\rangle$ and minimizes the uncertainty determined by the quadratic form in (7.1). A multi–mode coherent packet propagates semiclassically on $g_{\rm eff}$:

$$\langle\hat\varphi(x)\rangle_\alpha\approx\varphi_{\rm cl}(x),\qquad\big(\Box_{g_{\rm eff}}+m^2\big)\varphi_{\rm cl}=0, \tag{7.8}$$

with anisotropic group velocity $\mathbf{v}_g=\nabla_{\mathbf{k}}\omega(\mathbf{k},\theta)$.

Wigner function and Liouville drift. Define $W[\varphi,\pi]$ on phase space; in the weak–noise limit it obeys a quantum–Liouville (Moyal) equation that reduces to classical Liouville transport plus small $\hbar^2$ corrections. Dissipation/noise from (7.6) adds Fokker–Planck terms that drive pointer sectors toward stable eigendistributions (phase locking).

7.4 Born Weights as a Stabilization Limit (Damascus Interpretation)

Pointer sectors and energy partition. Let the instrument coupling diagonalize into orthogonal pointer sectors $\{\mathcal{H}_i\}$ with projectors Π_i, stabilized by large (ρ, η) (Sec. 1.5). Under the GKSL flow (7.7), coherences decay as $\|\Pi_i \rho\, \Pi_j\|_1 \sim e^{-t/\tau_\phi}$, $i \neq j$, while the *energy influx* into sector i obeys

$$\frac{d}{dt} E_i(t) = \mathrm{Tr}(\Pi_i\, \dot{\rho}\, H_{\mathrm{eff}}) \;\simeq\; \int d^3x\, \varepsilon_i(x) \;-\; \frac{1}{\tau_{\mathrm{lock}}} E_i(t), \tag{7.9}$$

with ε_i the *spectral* energy–density source induced by the system state in channel i and τ_{lock} the phase–locking time set by η and the instrument impedance.

Stationary limit and Born weights. For $t \gg \tau_\phi, \tau_{\mathrm{lock}}$ one finds a stationary partition

$$P_i \;\equiv\; \lim_{t\to\infty} \mathrm{Tr}(\Pi_i \rho_t) \;=\; \frac{\int d^3x\, \varepsilon_i(x)}{\sum_j \int d^3x\, \varepsilon_j(x)} \;=\; \int d^3x\, |\Psi_i(x)|^2, \tag{7.10}$$

where the last equality identifies $|\Psi_i|^2$ as the *normalized* spectral–energy density projected onto channel i.[1]

Deviation bounds. For finite stabilization,

$$\Big|P_i - \int |\Psi_i|^2\Big| \;\le\; \underbrace{\mathcal{O}\Big(\frac{\tau_\phi}{\tau_{\mathrm{lock}}}\Big)}_{\text{incomplete dephasing}} + \underbrace{\mathcal{O}\Big(\frac{\|H_{\mathrm{LS}}\|}{\gamma_{\min}}\Big)}_{\text{Lamb–shift bias}} + \underbrace{\mathcal{O}\Big(\frac{\eta_{\mathrm{res}}}{\eta_{\mathrm{instr}}}\Big)}_{\text{residual backaction}}, \tag{7.11}$$

with $\gamma_{\min}$ the smallest nonzero GKSL rate and H_{LS} the Lamb–shift correction. Complete positivity ensures $0 \le P_i \le 1$ and $\sum_i P_i = 1$ at all times.

7.5 Spectral Path Integral for Open Dynamics: Tests

Double slit with phase–only tagging. In the CTP framework (7.5)–(7.6), a phase–only tag near one slit modifies the interference term by

$$\mathcal{V}(k,\theta) \;=\; \exp\!\left[-\int d\omega\, \mathcal{J}(\omega)\, \frac{\sin^2(\omega\,\Delta t/2)}{(\omega/2)^2}\right], \quad \Delta\phi \sim \int dt\, \mathrm{Re}\,\mathsf{D}(t), \tag{7.12}$$

yielding measurable fringe–phase shifts at nearly fixed counts (diagnostic for flooding vs. instantaneous collapse).

Colored Zeno. For pulsed monitoring with spectral occupancy $n_{\mathrm{pulse}}(\omega)$, define the *colored Zeno parameter*

$$\mathcal{Z} = \int_0^\infty d\omega\, \mathcal{J}(\omega)\, n_{\mathrm{pulse}}(\omega), \qquad \Gamma_{\mathrm{eff}} = \Gamma_0 - \alpha\, \mathcal{Z} + \cdots, \tag{7.13}$$

showing Zeno/anti–Zeno crossovers by shaping n_{pulse} relative to the instrument spectral density $\mathcal{J}$.

[1] This reproduces the Born rule as a *hydrodynamic* fixed point of spectral flooding. No extra postulate is required.

7.6 Gates, Restoration, and Scope

- **(GQ1) Causality.** Noise/dissipation kernels vanish outside the g_{eff} cone; path integrals are defined with retarded support.
- **(GQ2) Complete positivity.** $\mathsf{N} \geq 0 \Rightarrow$ GKSL form (7.7); $P_i \in [0,1]$.
- **(GQ3) Entropy production.** For $\eta > 0$, $\dot{S}_{\text{vN}} \geq 0$ (cf. Vol. III, Sec. 1.4.4).
- **(GQ4) Restoration.** As $\eta \downarrow 0$ and coarse–graining increases, anisotropies $c_\alpha(\theta) \to c$ and the theory reduces to isotropic single–clock EFT with the standard (Born) measurement statistics as the stabilized limit.

Outcome. Within the Damascus interpretation, quantum probabilities are stabilized energy–share weights produced by spectral flooding under physically controlled dissipation. The spectral CTP/GKSL machinery ties this outcome to first principles (causality, CP, positivity), while coherent sectors and Wigner transport recover classical behavior on g_{eff}, completing the bridge from spectral ontology to operational quantum mechanics.

Chapter 8

Precision Calibration: Corrected QED and Atomic Spectra

Aim. Tie the spectral framework to precision laboratory data by deriving (i) the effective $U(1)$ electrodynamics on the emergent geometry g_{eff} with spectral counterterms, (ii) bound–state energies (Dirac–Coulomb, fine structure) on g_{eff}, (iii) Lamb and vacuum–polarization shifts including spectral loop/propagator corrections, (iv) anomalous magnetic moments a_e, a_μ from Pauli–type operators and spectral loops, (v) hyperfine structure and possible sidereal signatures, and (vi) a parameter map from spectral couplings to observables with conservative bounds. Throughout we enforce the S/G/TQ/D gates (stability, causality/hyperbolicity, quantum CP/positivity, dissipation nonnegativity).

8.1 Effective Electrodynamics on g_{eff} and Spectral Counterterms

The emergent $U(1)$ arises from isometries of the internal spectral fiber (Vol. I, Ch. 4; Vol. III, Sec. 1.9.1). The low–energy Lagrangian density (in units $\hbar = c = 1$ unless stated) on g_{eff} is

$$\begin{aligned}\mathcal{L}^{\text{eff}}_{\text{EM}} = &-\frac{Z_F}{4}\, g^{\mu\alpha}_{\text{eff}}\, g^{\nu\beta}_{\text{eff}} F_{\mu\nu}F_{\alpha\beta} \;-\; \frac{1}{4}\,(k_F)^{\mu\nu\alpha\beta} F_{\mu\nu}F_{\alpha\beta} \;+\; \frac{\theta}{4}\,\epsilon^{\mu\nu\alpha\beta} F_{\mu\nu}F_{\alpha\beta} \\ &+\; \bar{\psi}(iZ_\psi \gamma^\mu D_\mu - m_e)\,\psi \;-\; \frac{\kappa_P}{4m_e}\,\bar{\psi}\sigma^{\mu\nu}\psi\, F_{\mu\nu} \;+\; \xi_{\chi F}\,\chi^\mu \bar{\psi}\gamma^\nu\psi\, F_{\mu\nu} \;+\; \cdots ,\end{aligned} \tag{8.1}$$

where $F_{\mu\nu} = \nabla_\mu A_\nu - \nabla_\nu A_\mu$, $D_\mu = \nabla_\mu + ieA_\mu$, and dots denote higher–dimension operators suppressed by a spectral scale Λ_{spec}. Here Z_F, Z_ψ absorb wave–function renormalizations, $(k_F)^{\mu\nu\alpha\beta}$ encodes tiny birefringent/anisotropic corrections inherited from the internal director $\hat{\chi}$ (traceless double–dual structure), κ_P is a Pauli term, and $\xi_{\chi F}$ mixes the director with the electromagnetic bilinear. The running coupling is

$$\alpha_{\text{eff}}(\mu,\theta) = \frac{e^2(\mu)}{4\pi Z_F(\mu,\theta)}\Big[1 + \delta^{\text{spec}}_\alpha(\mu,\theta)\Big], \qquad \delta^{\text{spec}}_\alpha = \mathcal{O}\Big(\frac{k_F}{Z_F},\; \frac{\mu^2}{\Lambda^2_{\text{spec}}}\Big), \tag{8.2}$$

with a mild angular dependence through $\theta \equiv \angle(\mathbf{k}, \hat{\chi})$ that must satisfy statistical–isotropy bounds (Vol. II).

Gates. Positivity of the photon Hamiltonian demands $Z_F > 0$ and that the principal symbol of the field equations remain inside the g_{eff} cone (G/S). Complete positivity and no–signalling are ensured by using retarded kernels and GKSL coupling to instruments (TQ/D gates; Vol. III, Sec. 1.4).

8.2 Dirac–Coulomb Bound States on g_{eff}: Fine Structure

For a hydrogenic ion (nucleus charge Ze) in the aether frame u^μ, the one–electron Hamiltonian reads

$$H_D = \beta m_e + c\,\boldsymbol{\alpha}\cdot\big(\mathbf{p} - e\mathbf{A}\big) + V_C(r) \;+\; \delta H_{\mathrm{aniso}} + \delta H_{\mathrm{Pauli}} + \cdots\,, \qquad V_C(r) = -\frac{Z\alpha_{\mathrm{eff}}}{r}, \tag{8.3}$$

with c the universal front speed fixed by Vol. III, Sec. 3.4. Anisotropy enters through small (k_F)–induced birefringence and $\xi_{\chi F}$,

$$\delta H_{\mathrm{aniso}} \;=\; \frac{1}{2}\,\delta Z_{ij}\,\{p_i, p_j\} \;+\; \delta n_{ij}\,\frac{x_i x_j}{r^3}\,, \qquad \delta H_{\mathrm{Pauli}} = -\frac{\kappa_P}{2m_e}\,\mathbf{S}\cdot\mathbf{B}, \tag{8.4}$$

where $(\delta Z_{ij}, \delta n_{ij}) \propto (k_F)_{0i0j}$ in the static limit. To leading order, the relativistic energies are

$$E^{(0)}_{n\kappa} = m_e c^2 \left[1 + \frac{(Z\alpha_{\mathrm{eff}})^2}{\big(n - \delta_j\big)^2}\right]^{-1/2}, \qquad \delta_j \equiv j + \tfrac{1}{2} - \sqrt{\big(j+\tfrac{1}{2}\big)^2 - (Z\alpha_{\mathrm{eff}})^2}, \tag{8.5}$$

with fine–structure splitting $\Delta E_{\mathrm{FS}} \sim (Z\alpha_{\mathrm{eff}})^4 m_e c^2/n^3$ and tiny directional corrections $\Delta E_{\mathrm{aniso}} \sim \langle \delta H_{\mathrm{aniso}}\rangle = \mathcal{O}((k_F)\,(Z\alpha)^2 m_e)$ that are bounded by clock–comparison and spectroscopy.

8.3 Lamb Shift and Vacuum Polarization with Spectral Corrections

The Lamb shift ΔE_L combines electron self–energy and vacuum polarization. In our framework:

$$\Delta E_L(n\ell j) = \;\Delta E_{\mathrm{SE}} + \Delta E_{\mathrm{VP}} + \Delta E_{\mathrm{spec}}, \tag{8.6}$$

$$\Delta E_{\mathrm{SE}} = \frac{\alpha_{\mathrm{eff}}}{\pi}\,\frac{(Z\alpha_{\mathrm{eff}})^4}{n^3}\,m_e c^2\left[\ln\!\big((Z\alpha_{\mathrm{eff}})^{-2}\big) + A_{40}(\ell, j) + A_{60} + \cdots\right], \tag{8.7}$$

$$\Delta E_{\mathrm{VP}} = \langle \psi_{n\ell j}|\delta V_{\mathrm{Ueh}}(r)|\psi_{n\ell j}\rangle, \qquad \delta V_{\mathrm{Ueh}}(r) = -\frac{2Z\alpha_{\mathrm{eff}}^2}{3\pi r}\int_1^\infty du\, e^{-2m_e r u}\left(1 + \frac{1}{2u^2}\right)\frac{\sqrt{u^2-1}}{u^2}, \tag{8.8}$$

while spectral corrections enter via modifications of the photon/electron propagators and seagull–type vertices:

$$\Delta E_{\mathrm{spec}} = \frac{(Z\alpha)^4}{n^3}\,m_e c^2\Big[\underbrace{\delta Z_F}_{\text{photon renorm}} + \underbrace{\delta Z_\psi}_{\text{electron renorm}} + \underbrace{\tilde{k}_F(\hat{\chi})}_{\text{birefringence}} + \underbrace{\frac{\kappa_P}{m_e}\,\langle \mathbf{S}\cdot\mathbf{B}\rangle}_{\text{Pauli}} + \cdots\Big]. \tag{8.9}$$

The angular dependence in $\tilde{k}_F(\hat{\chi})$ averages out in isotropic samples; any residual sidereal/rotational modulation can be constrained by comparing $1S$–$2S$, $2S$–nD, and fine–structure intervals in different orientations.

8.4 Anomalous Magnetic Moments a_e and a_μ

Augmenting QED with the Pauli operator and spectral loop corrections yields

$$a_f \equiv \frac{g_f - 2}{2} = a_f^{\mathrm{QED}}(\alpha_{\mathrm{eff}}) + \underbrace{\frac{\kappa_P^{(f)}}{2}}_{\text{Pauli tree}} + \underbrace{\Delta a_f^{\mathrm{loop}}(k_F, \xi_{\chi F}, \Lambda_{\mathrm{spec}})}_{\text{spectral loops}} + \cdots, \qquad f = e, \mu. \tag{8.10}$$

Here a_f^{QED} is the standard perturbative series in α_{eff}, while $\Delta a_f^{\mathrm{loop}} \sim \mathcal{O}(m_f^2/\Lambda_{\mathrm{spec}}^2)$ up to small birefringent weights. Given the experimental precisions (electron at $\sim 10^{-12}$, muon at $\sim 10^{-9}$ level), we infer conservative priors

$$|\kappa_P^{(e)}| \lesssim 10^{-12}, \qquad |\kappa_P^{(\mu)}| + |\Delta a_\mu^{\mathrm{loop}}| \lesssim \text{few} \times 10^{-9},$$

to be refined by a joint fit with spectroscopy (Sec. 8.7).

8.5 Hyperfine Structure and Sidereal Signatures

The Fermi contact interaction for S–states,

$$\Delta E_{\mathrm{HFS}}^{\mathrm{Fermi}} = \frac{8}{3}\,\alpha_{\mathrm{eff}}^4\,\frac{m_r^3}{m_e m_p}\,c^2\,\frac{\mu_p}{\mu_N}\,\frac{1}{n^3}\,\delta_{\ell 0}, \tag{8.11}$$

receives spectral corrections from (k_F)–induced anisotropy and $\xi_{\chi F}$ that couple electron spin to a preferred direction $\hat{\chi}$:

$$\delta E_{\mathrm{HFS}}^{\mathrm{aniso}} = \lambda_\chi\,(\hat{\chi}\cdot\hat{\mathbf{B}})\,\langle S_\parallel\rangle + \mathcal{O}\big((k_F)\alpha^4\big), \qquad \lambda_\chi \propto \xi_{\chi F}\,B_{\mathrm{ext}}. \tag{8.12}$$

Clock–comparison and comagnetometer experiments bound sidereal modulations of $\Delta E_{\mathrm{HFS}}^{\mathrm{aniso}}$; we adopt tight priors on (k_F)–components and $\xi_{\chi F}$ in the parameter map below.

8.6 Rydberg Constant, α, and Consistency

Define the effective Rydberg constant

$$R_\infty^{\mathrm{eff}} = \frac{\alpha_{\mathrm{eff}}^2\,m_e c}{2h}, \qquad \frac{\delta R_\infty}{R_\infty} = 2\,\frac{\delta\alpha_{\mathrm{eff}}}{\alpha_{\mathrm{eff}}} + \frac{\delta m_e}{m_e} + \frac{\delta c}{c} - \frac{\delta h}{h}. \tag{8.13}$$

Within the spectral framework, $\delta c/c \to 0$ at low energies (Vol. III, Sec. 3.4) and h is fixed, so R_∞ calibrations constrain primarily $\delta\alpha_{\mathrm{eff}}$ and any tiny m_e renormalization from spectral loops. Combining $1S$–$2S$, $2S$–nD, ion–trap g–factor, and a_e yields a closed consistency triangle for α_{eff}.

8.7 Parameter Map and Conservative Constraints

We summarize the leading mappings from spectral effective couplings to precision observables. Bounds are indicative orders–of–magnitude to be replaced by a full fit (Part V, Inference Pipelines).

Spectral parameter	**Observable handle**	**Indicative bound**
$Z_F(\mu,\theta)$, $\delta_\alpha^{\rm spec}$	R_∞, $1S$–$2S$, global hydrogenic fit	$\lvert\delta\alpha/\alpha\rvert \lesssim 10^{-10}$ (isot
$(k_F)^{\mu\nu\alpha\beta}$ (birefringence/aniso.)	fine structure anisotropy; sidereal HFS	$\lvert(k_F)\rvert \lesssim 10^{-17}$–$10^{-20}$
$\kappa_P^{(e)}$ (Pauli)	a_e, bound–state g–factor	$\lvert\kappa_P^{(e)}\rvert \lesssim 10^{-12}$
$\kappa_P^{(\mu)}$, $\Delta a_\mu^{\rm loop}$	a_μ	$\lesssim$ few $\times\, 10^{-9}$
$\xi_{\chi F}$ (director–EM mixing)	HFS sidereal modulation; Zeeman anisotropy	$\lesssim 10^{-11}$–10^{-13} (per
m_e spectral renorm. (isotropic)	R_∞ vs. cyclotron g	$\lvert\delta m_e/m_e\rvert \lesssim 10^{-11}$

Table 8.1: Leading map from spectral couplings to precision QED observables. Numbers are conservative targets; the formal fit will adopt the joint likelihoods in Part V.

Gates recap. All fits are constrained by: (G) positivity/no–ghost for photon/electron sectors; (S) subluminal front velocities inside the $g_{\rm eff}$ cone; (TQ) complete positivity (no negative widths) and consistency of open dynamics; (D) nonnegative entropy production with $\eta > 0$. In practice, these gates define priors in the sampling pipelines (Vol. II, Part V).

Outcome. The emergent–$U(1)$ plus tiny spectral counterterms reproduce standard QED to current laboratory accuracy while providing concrete, multi–channel levers—Lamb, a_e, HFS, fine structure, and R_∞—to bound $(k_F, \kappa_P, \xi_{\chi F}, \delta_\alpha^{\rm spec})$. This closes the energy–momentum accounting in media (Poynting) and anchors the Damascus interpretation to precision metrology.

Chapter 9

Emergent Gauge Symmetry and BRST/FP Structure

Aim. Show how gauge symmetry arises from isometries of the internal spectral fiber, construct a gauge–fixed path integral on the emergent geometry g_{eff}, derive the Faddeev–Popov (FP) determinant and the BRST symmetry, and prove Ward/Slavnov–Taylor identities, gauge–parameter independence (Nielsen identities), and unitarity in the physical subspace. We also state GKSL–compatible conditions ensuring that open–system (dissipative) effects preserve gauge covariance and no–signalling.

9.1 From Fiber Isometries to Gauge Orbits

Let $\mathcal{F}$ be the internal spectral fiber with metric $\mathcal{G}_{IJ}(y)$ and isometry group G acting as $y^I \mapsto g\cdot y^I$. A configuration $y^I(x)$ defines a principal G–bundle $\pi : \mathcal{P} \to \mathcal{M}$ (base $\mathcal{M}$ is the effective spacetime endowed with g_{eff}). Define a local section $U(x) \in G$ such that the Maurer–Cartan form projects onto the Lie algebra $\mathfrak{g}$:

$$A_\mu(x) \;\equiv\; \mathsf{P}_{\mathfrak{g}}\Big[\,U^{-1}(x)\,\partial_\mu U(x)\,\Big] \;\in\; \mathfrak{g}, \qquad F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + [A_\mu, A_\nu]. \tag{9.1}$$

Under a local isometry $g(x) \in G$,

$$A_\mu \;\mapsto\; A^g_\mu \;=\; g^{-1}A_\mu g + g^{-1}\partial_\mu g, \qquad F_{\mu\nu} \mapsto g^{-1}F_{\mu\nu}g, \tag{9.2}$$

so the connection A_μ and curvature $F_{\mu\nu}$ are *emergent gauge fields*. Matter/structural fields Ψ transform in some representation R with covariant derivative $D_\mu\Psi = \nabla_\mu\Psi + g\,A^a_\mu T^a_R\Psi$ (here ∇_μ is Levi–Civita on g_{eff}).

The gauge–invariant Yang–Mills sector on $(\mathcal{M}, g_{\text{eff}})$ is

$$\mathcal{L}_{\text{YM}} \;=\; -\frac{1}{4}\,Z_F\,g^{\mu\alpha}_{\text{eff}}g^{\nu\beta}_{\text{eff}}\,F^a_{\mu\nu}F^a_{\alpha\beta} \;+\; \bar\Psi(iZ_\Psi\gamma^\mu D_\mu - m)\Psi \;+\; \cdots, \tag{9.3}$$

where dots denote higher–dimension operators suppressed by the spectral scale Λ_{spec} and tiny anisotropic corrections consistent with the S/G gates.

9.2 Gauge Fixing on $g_{\rm eff}$ and the FP Determinant

Choose a covariant gauge functional $\mathcal{G}^a[A] = \nabla^\mu A^a_\mu$ with respect to $g_{\rm eff}$. In the path integral $Z = \int \mathcal{D}A\,\mathcal{D}\Psi\, e^{i\int \sqrt{-g_{\rm eff}}\mathcal{L}}$, insert unity as

$$1 = \Delta_{\rm FP}[A] \int \mathcal{D}g\ \delta(\mathcal{G}^a[A^g])\,, \qquad \Delta_{\rm FP}[A] = \det\!\Big[-\nabla^\mu D^{ab}_\mu(A)\Big]\,, \tag{9.4}$$

where $D^{ab}_\mu = \delta^{ab}\nabla_\mu + g f^{acb} A^c_\mu$. Exponentiating the determinant with ghost fields $(c^a, \bar c^a)$ and introducing the Nakanishi–Lautrup field B^a yields the gauge–fixed Lagrangian

$$\mathcal{L}_{\rm gf+FP} = \sqrt{-g_{\rm eff}}\left[-\frac{1}{2\xi}\,(\nabla^\mu A^a_\mu)^2 \;+\; i\,\bar c^a\,\nabla^\mu D^{ab}_\mu c^b \;+\; B^a\,\mathcal{G}^a[A] \;+\; \frac{\xi}{2}\,B^a B^a\right]. \tag{9.5}$$

For Abelian $G = U(1)$ the ghosts decouple ($D_\mu = \nabla_\mu$), but we retain them to keep the BRST algebra uniform.

9.3 BRST Symmetry and Nilpotency

Define the (Grassmann–odd) BRST operator s acting on fields as

$$s\,A^a_\mu = D^{ab}_\mu c^b, \qquad s\,c^a = -\tfrac{g}{2} f^{abc} c^b c^c, \qquad s\,\bar c^a = i\,B^a, \qquad s\,B^a = 0, \qquad s\,\Psi = ig\,c^a T^a_R \Psi, \tag{9.6}$$

with $s^2 = 0$ (nilpotent) by the Jacobi identity. The total Lagrangian $\mathcal{L}_{\rm tot} = \mathcal{L}_{\rm YM} + \mathcal{L}_{\rm gf+FP}$ is BRST–invariant up to a total derivative:

$$\mathcal{L}_{\rm gf+FP} \;=\; s\Big[\,\sqrt{-g_{\rm eff}}\,\bar c^a\Big(\mathcal{G}^a[A] - \tfrac{\xi}{2}B^a\Big)\Big]\,. \tag{9.7}$$

Physical states lie in the cohomology $\mathcal{H}_{\rm phys} = \ker Q_{\rm BRST}/\mathrm{Im}\,Q_{\rm BRST}$, where $Q_{\rm BRST}$ is the conserved BRST charge; unphysical polarizations cancel against ghosts in unitarity cuts (Kugo–Ojima mechanism).

9.4 Ward/Slavnov–Taylor and Gauge–Parameter Independence

Introduce sources (K^a_μ, L^a) for the composite BRST variations (sA^a_μ, sc^a). The 1PI functional Γ satisfies the Slavnov–Taylor identity

$$\mathcal{S}(\Gamma) \;\equiv\; \int d^4x \sqrt{-g_{\rm eff}}\left(\frac{\delta\Gamma}{\delta A^{a\mu}}\frac{\delta\Gamma}{\delta K^a_\mu} + \frac{\delta\Gamma}{\delta c^a}\frac{\delta\Gamma}{\delta L^a} + B^a\frac{\delta\Gamma}{\delta \bar c^a}\right) = 0. \tag{9.8}$$

From $\nabla_\mu \langle J^{\mu a}\rangle = 0$ and (9.8) follow the usual Ward identities, including the transversality of the gauge polarization tensor,

$$k_\mu\,\Pi^{\mu\nu}_{ab}(k) \;=\; 0, \tag{9.9}$$

which encodes charge conservation on $(\mathcal{M}, g_{\rm eff})$.

Gauge–parameter independence of on–shell observables is expressed by the Nielsen identity. For the effective action,

$$\frac{\partial\Gamma}{\partial\xi} \;+\; \int d^4x \sqrt{-g_{\rm eff}}\,\mathcal{C}^i(\Phi,\xi)\,\frac{\delta\Gamma}{\delta\Phi^i} \;=\; 0, \tag{9.10}$$

with Φ^i denoting all fields and $\mathcal{C}^i$ computable from BRST sources. Hence $\partial_\xi \Gamma|_{\rm on\ shell} = 0$ and the S–matrix is ξ–independent.

9.5 Background Field Method and Running Couplings

Split $A_\mu = \bar{A}_\mu + a_\mu$ and gauge–fix covariantly with respect to $\bar{A}_\mu$:

$$\mathcal{L}_{\rm gf}^{\rm BFM} \;=\; -\frac{1}{2\xi}\,(\bar{D}^\mu a_\mu^a)^2, \qquad \bar{D}_\mu^{ab} = \delta^{ab}\nabla_\mu + g f^{acb}\bar{A}_\mu^c. \tag{9.11}$$

Background gauge invariance $\bar{A}_\mu \mapsto \bar{A}_\mu^g$ is preserved, implying

$$Z_g\, Z_{\bar{A}}^{1/2} = 1, \qquad \beta(g) = \mu\,\frac{dg}{d\mu} = -\frac{g}{2}\,\mu\,\frac{d\ln Z_{\bar{A}}}{d\mu}. \tag{9.12}$$

This facilitates the map from spectral couplings to observed running (Part V) and ensures simple Ward identities for background Green's functions on $g_{\rm eff}$.

9.6 GKSL–Compatible Dissipation and Modified Ward Identities

Open–system effects appear through GKSL generators (Vol. III, Sec. 1.4). Gauge covariance and complete positivity require Lindblad operators in *gauge–covariant* combinations (e.g. $L \sim F_{\mu\nu}$, $D_\mu\Psi$) and retarded kernels with support inside the $g_{\rm eff}$ cone (S/G/TQ gates). The polarization tensor decomposes as

$$\Pi_{ab}^{\mu\nu}(k) \;=\; \Pi_T(k)\,\mathcal{P}_T^{\mu\nu}(k)\,\delta_{ab} \;+\; i\,\Pi_D(k)\,\epsilon^{\mu\nu\alpha\beta}u_\alpha\hat{k}_\beta\,\delta_{ab} \;+\; \cdots, \tag{9.13}$$

with $k_\mu\Pi^{\mu\nu} = 0$ maintained and $\mathrm{Im}\,\Pi_T(k) \geq 0$ (Kubo positivity). Thus dissipation broadens propagators but does not violate charge conservation nor induce gauge–variant mass terms.

9.7 Gates and Unitarity in the Physical Subspace

- **(G) Stability:** Positive–definite kinetic blocks for $(A_\mu, \bar{c}, c)$ with principal cones inside that of $g_{\rm eff}$; no tachyonic instabilities in the gauge sector.
- **(S) Causality/Hyperbolicity:** Front velocities bounded by c of $g_{\rm eff}$; gauge choices (Lorenz/BFM) preserve hyperbolicity on $(\mathcal{M}, g_{\rm eff})$.
- **(TQ) Quantum consistency:** BRST nilpotency, Slavnov–Taylor identities, and complete positivity of open dynamics ⇒ unitary S–matrix on $\mathcal{H}_{\rm phys}$ (Kugo–Ojima).
- **(D) Dissipation:** $\eta > 0$ yields nonnegative entropy production; GKSL kernels are gauge–covariant and retarded, ensuring no–signalling.

Outcome. Gauge symmetry in the spectral framework is a redundancy inherited from fiber isometries. The standard BRST/FP machinery extends cleanly to the emergent geometry $g_{\rm eff}$, with Ward/Slavnov–Taylor and Nielsen identities guaranteeing gauge–parameter independence and unitarity of physical observables. Dissipative (GKSL) effects compatible with gauge covariance modify widths but preserve the transverse structure and no–signalling, closing the gauge–consistency loop needed for precision and collider–level predictions.

Chapter 10

Topological Anomalies and Inflow: ABJ, 't Hooft, and WZW/CS

Aim. Catalogue and control topological anomalies in the emergent gauge sector (Sec. 9): axial ABJ anomalies, perturbative gauge/mixed/gravitational anomalies, and global 't Hooft anomalies. We formulate the anomaly polynomial and descent on the emergent geometry g_{eff}, derive the Adler–Bell–Jackiw (ABJ) result via Fujikawa's method, state and use cancellation/matching conditions, and exhibit Wess–Zumino–Witten (WZW) and Chern–Simons (CS) terms generated by anomaly inflow. Throughout, S/G/TQ/D gates are enforced, and open–system (GKSL) effects are constrained to preserve anomaly (co)variance and no–signalling.

10.1 Anomaly Polynomial and Descent on g_{eff}

Consider a chiral fermion in representation R of a compact gauge group G, coupled to the emergent connection A_μ on the effective spacetime $(\mathcal{M}, g_{\text{eff}})$. The consistent gauge anomaly in $d=4$ arises from a closed $(d+2)$–form (the anomaly polynomial) in $D=6$:

$$\mathcal{I}_6 \;=\; \Big[\hat{A}(R_{\text{eff}})\,\mathrm{ch}(F)\Big]_6 \;=\; \frac{1}{6}\,\mathrm{tr}_R(F^3) \;-\; \frac{1}{24}\,\mathrm{tr}_R(F)\,p_1(T\mathcal{M}), \tag{10.1}$$

where F is the field strength two–form, $p_1(T\mathcal{M}) = \frac{1}{8\pi^2}\mathrm{tr}(R\wedge R)$ is the first Pontryagin class of the Levi–Civita curvature R of g_{eff}, and $\hat{A}$ and ch are the Dirac genus and Chern character. The descent equations

$$\mathcal{I}_6 = d\,\mathcal{I}_5^{(0)}, \qquad \delta_\alpha \mathcal{I}_5^{(0)} = d\,\mathcal{I}_4^{(1)}(\alpha), \tag{10.2}$$

imply a non–invariance of the effective action $\delta_\alpha \Gamma[A, g_{\text{eff}}] \;=\; 2\pi i \int_{\mathcal{M}} \mathcal{I}_4^{(1)}(\alpha)$, with α the gauge parameter. The *covariant* anomaly differs from the consistent one by a Bardeen–Zumino polynomial; both are controlled by $\mathcal{I}_6$.

S/G gates. Hyperbolicity/causality on g_{eff} ensures well–posed Dirac dynamics; the anomaly is regulator–robust provided the regulator preserves gauge covariance and g_{eff}–causality. Positivity (TQ gate) constrains admissible counterterms (no negative–norm "fixes").

10.2 ABJ Anomaly on g_{eff} (Fujikawa)

For a single Dirac fermion with vector and axial currents $J^\mu = \bar\psi\gamma^\mu\psi,\ J_5^\mu = \bar\psi\gamma^\mu\gamma_5\psi$, a chiral rotation $\psi \to e^{i\alpha\gamma_5}\psi$ changes the fermion measure by

$$\mathcal{J} \;=\; \exp\Big\{ \frac{i}{16\pi^2}\int d^4x\sqrt{-g_{\text{eff}}}\,\alpha(x)\,\Big[\mathrm{tr}(F_{\mu\nu}\tilde F^{\mu\nu}) - \tfrac{1}{12}\,\mathrm{tr}(R_{\mu\nu\rho\sigma}\tilde R^{\mu\nu\rho\sigma})\Big]\Big\}, \tag{10.3}$$

yielding the ABJ non–conservation

$$\nabla_\mu J_5^\mu \;=\; \frac{1}{16\pi^2}\,\mathrm{tr}(F_{\mu\nu}\tilde F^{\mu\nu}) \;-\; \frac{1}{192\pi^2}\,\mathrm{tr}(R_{\mu\nu\rho\sigma}\tilde R^{\mu\nu\rho\sigma}), \tag{10.4}$$

with duals $\tilde F^{\mu\nu} = \frac{1}{2}\varepsilon^{\mu\nu\alpha\beta}F_{\alpha\beta}$ and $\tilde R^{\mu\nu\rho\sigma} = \frac{1}{2}\varepsilon^{\rho\sigma\alpha\beta}R^{\mu\nu}{}_{\alpha\beta}$ defined using g_{eff}. The result depends only on the topology (index of the Dirac operator) and is insensitive to small anisotropies or GKSL–admissible damping.

10.3 Perturbative Gauge/Mixed/Gravitational Anomalies and Cancellation

For a chiral fermion content $\{\psi_i\}$ in representations R_i with $U(1)$ charges q_i, the 4D anomaly constraints read

$$\text{[pure non–Abelian]} \qquad \sum_i \mathrm{Tr}_{R_i}(T^a\{T^b,T^c\}) \;=\; 0, \tag{10.5}$$

$$\text{[mixed } G^2\text{–}U(1)\text{]} \qquad \sum_i q_i\,\mathrm{Tr}_{R_i}(T^aT^b) \;=\; 0, \qquad \text{[pure } U(1)^3\text{]} \quad \sum_i q_i^3 \;=\; 0, \tag{10.6}$$

$$\text{[mixed } U(1)\text{–grav}^2\text{]} \qquad \sum_i q_i \;=\; 0. \tag{10.7}$$

Here T^a are Lie algebra generators in R_i, with standard trace normalizations.

Boxed condition (spectral SM template). For an emergent $SU(3)\times SU(2)\times U(1)_Y$ content $\{\mathfrak{R}_i, Y_i\}$,

$$\sum_i Y_i = \sum_i Y_i^3 = \sum_i Y_i\,\mathrm{Tr}_{\mathfrak{R}_i}(T^aT^b) = \sum_i \mathrm{Tr}_{\mathfrak{R}_i}(T^a\{T^b,T^c\}) = 0,$$

must hold generation by generation (or in total), fixing hypercharge normalization and forbidding anomalous solitonic assignments. These constraints map the *spectral charges* of excitations to their effective gauge quantum numbers.

Bardeen counterterm. Vector current conservation within a mixed V–A setup is ensured by a Bardeen counterterm; physical observables are independent of the "consistent vs covariant" choice provided the above sums vanish.

10.4 't Hooft Anomaly Matching for Global Symmetries

Let $\mathcal{G}_{\text{global}}$ be a global (possibly discrete or higher–form) symmetry of the microscopic spectral theory. If its 't Hooft anomaly is nonzero in the UV (computed from the UV

matter content/solitons), then any IR phase (confining, Higgs, topological) must *match* the anomaly—by massless degrees of freedom, Goldstone WZW terms, topological order, or anomaly inflow from a higher–dimensional bulk:

$$\mathcal{I}_{d+2}^{\mathrm{UV}} \;\equiv\; \mathcal{I}_{d+2}^{\mathrm{IR}} \quad \text{in cohomology.} \tag{10.8}$$

Consequence for spectral composites: any proposed binding pattern of chiral solitons into massive fermions must preserve the net anomaly of $\mathcal{G}_{\mathrm{global}}$ (e.g. axial/flavor/center symmetries), constraining allowable IR textures and defect condensates.

10.5 WZW and CS Terms from Anomaly Inflow

Wess–Zumino–Witten (4D). For a spontaneously broken global symmetry $G_L \times G_R \to G_V$ with Goldstone field $U(x) \in G$, the unique local functional reproducing the chiral anomaly is the WZW action:

$$S_{\mathrm{WZW}}[U, A_L, A_R] \;=\; 2\pi\, i \int_{B^5} \mathcal{I}_5^{(0)}(U, A_L, A_R), \qquad \partial B^5 = \mathcal{M}_4, \tag{10.9}$$

whose variation gives the consistent anomaly via descent. In the spectral medium, U parametrizes phases of composite excitations (e.g. flavor solitons); gauge couplings enter through the emergent connections $A_{L/R}$.

Chern–Simons (odd d). On odd–dimensional defects/domain walls or in effectively 3D phases, gauge invariance up to boundary terms requires CS actions

$$S_{\mathrm{CS}}[A] \;=\; \frac{k}{4\pi} \int_{\mathcal{M}_3} \mathrm{tr}\Big(A dA + \tfrac{2}{3} A^3\Big), \qquad k \in \mathbb{Z} \text{ (for compact simple } G), \tag{10.10}$$

with level quantization set by large gauge transformations. Inflow from a $(d+1)$–bulk cancels boundary anomalies. An axion–like density $\theta\, F\tilde{F}$ in 4D similarly accounts for parity–odd responses (e.g. chiral magnetic effect) with θ a spectral phase.

Parity / 3D "parity anomaly". For 3D fermions on spectral interfaces, an induced half–integer CS level signals spectral flow; consistency demands either doubled species or inflow that restores large–gauge invariance.

10.6 Gates and GKSL Compatibility

- **(S) Causality/hyperbolicity.** Regulators and GKSL kernels must be retarded and supported inside the cone of g_{eff}; anomaly coefficients are topological and regulator–independent.
- **(G) Gauge consistency.** Perturbative gauge anomalies must *cancel*. Residual global anomalies must be matched ('t Hooft) by IR content or inflow terms. CS/WZW coefficients obey quantization consistent with large gauge transformations.
- **(TQ) Quantum positivity/unitarity.** Physical S–matrix remains unitary in the BRST cohomology; dissipative widths ($\mathrm{Im}\,\Pi \geq 0$) do not alter anomaly polynomials.
- **(D) Dissipation.** GKSL generators may broaden correlators but cannot remove a true (global) anomaly; they may shift between consistent/covariant forms via Bardeen–Zumino–like local functionals without changing cohomology.

10.7 Diagnostics and Phenomenology

- $\pi^0 \to \gamma\gamma$ **and axial couplings.** WZW fixes the $\pi^0 F\tilde{F}$ vertex; in the spectral picture, the coefficient is tied to the integer winding of the composite phase U.
- **Electroweak anomalies.** Mixed $SU(2)^2-U(1)$ and $U(1)^3$ constraints calibrate hypercharge assignments of spectral excitations and forbid anomalous soliton spectra.
- **Topological responses.** Axion–like $\theta\, F\tilde{F}$ or 3D CS terms predict quantized Hall–type responses on defects/interfaces in anisotropic spectral phases.
- **Lattice/continuum checks.** Spectral–lattice Dirac operators with Ginsparg–Wilson property realize exact index theorems; numerical spectral flow across background instantons verifies (10.4) on g_{eff}.

Outcome. With anomaly polynomials/descent formulated on the emergent geometry, ABJ is recovered, perturbative gauge anomalies are cancelled by representation constraints, and global 't Hooft anomalies are matched via IR content or inflow. WZW/CS terms appear naturally as boundary functionals of the spectral phase space, with quantized coefficients. The S/G/TQ/D gates restrict admissible dissipative deformations, ensuring that anomaly–driven phenomena remain coherent and that the emergent gauge sector is microscopically consistent.

Chapter 11

Strong Sector: Confinement and Chiral Symmetry Breaking

Aim. Build an explicit bridge from the spectral medium $\mathcal{S}(x) = \{\Phi, \rho, \eta, \chi^\mu, T\}$ to hadronic phenomena: asymptotically–free color dynamics at short distances, confinement and string–like flux at intermediate scales, and spontaneous chiral symmetry breaking (SχSB) with pions as Nambu–Goldstone bosons and baryons as topological solitons. We present (i) an emergent $SU(3)_c$ gauge sector, (ii) a chiral effective theory with WZW term, (iii) Faddeev–Skyrme solitons as baryons/glueballs, (iv) trace–anomaly/dilaton dynamics, and (v) Gell–Mann–Oakes–Renner (GOR) relations, all under S/G/TQ/D gates.

11.1 Emergent Color and Confinement Diagnostics

Color as fiber isometry. Let $\mathcal{F}_{\rm int}$ be the internal spectral fiber of Vol. I; assume an isometry subgroup $\mathrm{Iso}(\mathcal{F}_{\rm int}) \supset SU(3)_c$. The corresponding gauge connection $A_\mu = A_\mu^a T^a$ and curvature $F_{\mu\nu}$ arise from the pullback by D_μ (spectral derivative):

$$F_{\mu\nu} \;=\; \partial_\mu A_\nu - \partial_\nu A_\mu + [A_\mu, A_\nu], \qquad \mathcal{L}_{\rm YM} \;=\; -\frac{1}{2}\,\mathrm{tr}\!\left(\frac{1}{g_s^2(\mathcal{S})}\,F_{\mu\nu}F^{\mu\nu}\right), \tag{11.1}$$

with an *effective* coupling $g_s(\mathcal{S})$ determined by the state (ρ, η, χ^μ) via the spectral kinetic metric (Vol. I, Ch. 3).

Asymptotic freedom and RG. Spectral coarse–graining yields a one–scale running $\mu\,\frac{d}{d\mu}\,\alpha_s(\mu) = \beta(\alpha_s) \simeq -b_0\alpha_s^2 + \cdots$ with $b_0 > 0$ for $SU(3)$ with n_f light flavors. Dimensional transmutation sets $\Lambda_{\rm QCD}$ as an RG invariant built from $\{g_s(\mathcal{S}), M_{\rm eff}\}$.

Confinement probes. Introduce a *spectral dielectric* $\varepsilon_c(\mathcal{S})$ for color, entering the Wilson loop:

$$\langle W[C]\rangle \;\sim\; \exp[-\,\sigma(\mathcal{S})\,\mathcal{A}[C]]\,, \qquad \sigma(\mathcal{S}) \;\equiv\; \frac{1}{\varepsilon_c(\mathcal{S})}\,\Lambda_{\rm QCD}^2. \tag{11.2}$$

Area law ($\sigma > 0$) follows when the medium enters a dual–Meissner phase: monopole–like spectral defects condense, expelling color flux into tubes. The center symmetry $\mathbb{Z}_3$ order parameter (Polyakov loop) is governed by an effective potential $V[L]$ whose coefficients are functions of (ρ, η, T).

S/G gates. Hyperbolicity on $g_{\rm eff}$ ensures causal propagation; positivity of Yang–Mills kinetic block prevents ghosts; GKSL dissipation (when present) must be adjoint–covariant and retarded, preserving no–signalling and confinement diagnostics.

11.2 Chiral Symmetry and the Low–Energy Effective Action

In the light–quark sector (u, d, s), the microscopic spectral couplings induce an approximate global $G = SU(3)_L \times SU(3)_R$ symmetry of the color–singlet composites. SχSB occurs, $G \to H = SU(3)_V$, with Goldstone field $U(x) \in SU(3)$.

Leading chiral Lagrangian (on $g_{\rm eff}$).

$$\mathcal{L}_\chi \;=\; \frac{f_\pi^2}{4}\,\mathrm{Tr}\Big(D_\mu U\, D^\mu U^\dagger\Big) \;+\; \frac{f_\pi^2 B_0}{2}\,\mathrm{Tr}\Big(M_q\,U^\dagger + U\,M_q^\dagger\Big) \;+\; \mathcal{L}_{\rm WZW} \;+\; \mathcal{L}_{\rm Skyrme} + \cdots\,, \tag{11.3}$$

where $M_q = \mathrm{diag}(m_u, m_d, m_s)$, D_μ uses the emergent $g_{\rm eff}$ connection, and f_π, B_0 are *spectral* low–energy constants (LECs) computable as integrals of correlators of Φ, ρ, η. The Wess–Zumino–Witten term $\mathcal{L}_{\rm WZW}$ is fixed by the anomaly (Ch. 10) with coefficient N_c, ensuring correct $\pi^0 \to \gamma\gamma$.

Skyrme stabilization. An $\mathcal{O}(p^4)$ term controlled by the spectral stiffness stabilizes baryons:

$$\mathcal{L}_{\rm Skyrme} \;=\; \frac{\kappa_{\rm Sk}}{32}\,\mathrm{Tr}\Big([U^\dagger D_\mu U,\, U^\dagger D_\nu U]^2\Big)\,, \qquad \kappa_{\rm Sk} = \kappa_{\rm Sk}(\rho, \eta, \chi^\mu). \tag{11.4}$$

GOR relation (preview). At tree level, $m_\pi^2 f_\pi^2 = (m_u + m_d)\,|\langle\bar{q}q\rangle|$, with $|\langle\bar{q}q\rangle| = f_\pi^2 B_0$ (derived in Sec. 11.5).

11.3 Baryons as Skyrmions and Faddeev–Skyrme Knots

Topological charge. For $U \in SU(2) \subset SU(3)$,

$$B \;=\; \frac{1}{24\pi^2}\int d^3x\,\varepsilon^{ijk}\,\mathrm{Tr}\Big[(U^\dagger\partial_i U)(U^\dagger\partial_j U)(U^\dagger\partial_k U)\Big] \;\in\; \mathbb{Z}, \tag{11.5}$$

identified with baryon number via anomaly matching (WZW term).

Skyrmion mass and scaling. With the hedgehog ansatz $U(\mathbf{x}) = \exp[i\hat{\mathbf{r}}\cdot\tau F(r)]$, the static energy is

$$E_{\rm Sky} \;=\; 4\pi\int_0^\infty dr\,\left\{\frac{f_\pi^2}{2}\left(F'^2 + \frac{2\sin^2 F}{r^2}\right) + \frac{\kappa_{\rm Sk}}{2}\frac{\sin^2 F}{r^2}\left(2F'^2 + \frac{\sin^2 F}{r^2}\right)\right\}. \tag{11.6}$$

Minimization yields $M_N \simeq E_{\rm Sky}$ (plus rotational/isospin quantization). Spectral anisotropy induces small splittings via $c_{\parallel,\perp}$ in D_i; S/G gates bound these splittings (see Sec. 11.8).

Faddeev–Skyrme knots (glueball/di–baryon candidates). For a unit vector $\mathbf{n}(x) \in S^2$ (a spectral director composite), the Faddeev–Skyrme functional

$$E_{FS}[\mathbf{n}] \;=\; \int d^3x \,\Big[\alpha\,(\partial_i \mathbf{n})^2 + \beta\,(\mathbf{n}\cdot(\partial_i \mathbf{n}\times\partial_j \mathbf{n}))^2\Big]\,, \tag{11.7}$$

supports Hopf solitons labeled by $Q_H \in \pi_3(S^2) = \mathbb{Z}$, with mass scaling $E_{FS} \sim |Q_H|^{3/4}$; spectral couplings $\alpha(\rho,\eta)$, $\beta(\rho,\eta)$ determine core sizes. Such knots model glueballs or hybrid states stabilized by spectral tension.

11.4 Trace Anomaly, Dilaton, and Scalar Sector

Trace anomaly on $g_{\rm eff}$. In the chiral limit,

$$T^{\mu}{}_{\mu} \;=\; \frac{\beta(g_s)}{2g_s}\,\mathrm{tr}(F_{\mu\nu}F^{\mu\nu}) \;+\; \mathcal{O}(m_q\bar{q}q). \tag{11.8}$$

Introduce a spectral dilaton φ (a ρ–like scalar) encoding broken scale invariance with potential $V(\varphi)$ chosen to reproduce (11.8) at the minimum:

$$\mathcal{L}_{\rm dil} \;=\; \frac{1}{2}(\partial\varphi)^2 - V(\varphi), \qquad V(\varphi) \;=\; \frac{\kappa_\varphi}{4}\,\varphi^4\left(\ln\frac{\varphi}{f_\varphi} - \frac{1}{4}\right) + \cdots\,. \tag{11.9}$$

Coupling U to φ via $(f_\pi \to f_\pi\,\varphi/f_\varphi)$ implements (approximate) scale–chiral symmetry, yielding an isoscalar scalar that interpolates to the f_0–like channel.

11.5 GOR Relations from the Spectral Medium

Let $\Sigma \equiv -\langle\bar{q}q\rangle$ denote the chiral condensate extracted from two–point spectral correlators of axial/vector currents. Varying $\mathcal{L}_\chi$ with respect to M_q at $U = \mathbf{1}$ gives

$$\Sigma \;=\; f_\pi^2 B_0 \;+\; \mathcal{O}(m_q). \tag{11.10}$$

Expanding the mass term in (11.3) to quadratic order in pion fields yields

$$m_\pi^2 f_\pi^2 \;=\; (m_u + m_d)\,\Sigma \;+\; \mathcal{O}(m_q^2), \tag{11.11}$$

the GOR relation. In the spectral picture, Σ is a functional of (ρ,η,Φ); mild anisotropies correct f_π by $f_\pi^2 \to \langle f_\pi^2(\theta)\rangle_\theta$ with bounded angular spread (gated by CMB/BBN/(c_T-1) constraints).

11.6 Hadron Spectrum: Scaling and Benchmarks

Vector mesons and hidden local symmetry (HLS). Introduce V_μ as an emergent gauge boson of hidden $SU(2)_V$:

$$\mathcal{L}_{\rm HLS} \;=\; -\frac{1}{2}\mathrm{Tr}(V_{\mu\nu}V^{\mu\nu}) + \frac{a\,f_\pi^2}{4}\,\mathrm{Tr}\Big((D_\mu\xi_L\,\xi_L^\dagger - D_\mu\xi_R\,\xi_R^\dagger)^2\Big)\,, \tag{11.12}$$

with $U = \xi_L^\dagger\xi_R$. The KSRF–like relation $m_\rho^2 \simeq a g_\rho^2 f_\pi^2$ emerges; a, g_ρ inherit spectral running. Glueballs follow from dilaton/Hopf sectors (Secs. 11.4–11.3).

Baryon observables. Quantizing collective rotations yields M_N, Δ splitting and axial charge g_A; small spectral anisotropy shifts moments of inertia by $\delta\mathcal{I}/\mathcal{I} \ll 1$. Calibrate LECs against lattice/exp. benchmarks (string tension σ, f_π, m_ρ, m_N).

11.7 Confinement at Finite T and the Center Symmetry

Polyakov loop potential. In the aether frame, the one–loop–improved effective potential for the Polyakov loop L reads

$$\frac{V[L]}{T^4} \;=\; -\,a_2(\mathcal{S},T)\,|L|^2 - \,a_3(\mathcal{S},T)\,\left(L^3+L^{*3}\right) + \,a_4(\mathcal{S},T)\,|L|^4 + \cdots\,, \tag{11.13}$$

with coefficients depending on (ρ,η) through $g_s(\mathcal{S},T)$ and dielectric effects; a first–order or crossover transition follows depending on n_f. The quark condensate $\Sigma(T)$ couples to L through a spectral Yukawa, capturing deconfinement–chiral interplay.

11.8 Gates, Positivity, and Open–System Effects

- **(S) Causality/hyperbolicity.** Yang–Mills and chiral modes propagate within the cone of g_{eff}; vector/tensor speeds satisfy $0 < c_s^2, c_T^2 \le 1$, with anisotropy bounded by early–universe gates.
- **(G) Gauge/chiral consistency.** Local gauge anomalies cancel (Ch. 10); WZW coefficient equals N_c; 'th Hooft matching holds across phases. Skyrme/dilaton coefficients respect positivity constraints of forward amplitudes.
- **(TQ) Unitarity/positivity.** Spectral sum rules (Källen–Lehmann) apply to color–singlet currents; open–system widths $\Gamma \ge 0$ (GKSL) broaden but do not violate optical theorems.
- **(D) Dissipation.** GKSL kernels in hadronic media are retarded and center–symmetric; they may damp correlators but cannot destroy topological charges (baryon number, Hopf charge).

11.9 Phenomenological Program

1. **LEC extraction.** Compute $(f_\pi, B_0, \kappa_{\text{Sk}}, g_\rho, \ldots)$ from spectral correlators (two–point functions of vector/axial currents); match to lattice and experiment.
2. **Baryon/glueball spectroscopy.** Minimize (11.6), (11.7) with g_{eff}–covariant derivatives; include rotational quantization and small anisotropy.
3. **GOR and chiral logs.** Verify (11.11) and one–loop corrections; map spectral dependence of Σ on (ρ,η).
4. **Flux tubes and string tension.** Evaluate $\sigma(\mathcal{S})$ from classical color–dielectric profiles; compare to lattice σ.

5. **Finite–T crossover.** Fit $V[L]$ coefficients to lattice EoS; predict correlations between deconfinement and spectral observables (e.g. viscosity η).

Outcome. Color gauge fields emerge as fiber isometries with asymptotic freedom, confinement is encoded by a spectral dielectric and center symmetry, chiral symmetry breaks dynamically yielding the standard chiral Lagrangian with WZW term, baryons arise as Skyrmions stabilized by a spectral Skyrme term, glueball–like excitations appear as Faddeev–Skyrme knots/dilaton modes, and GOR relations follow from the spectral condensate. All elements are consistent with S/G/TQ/D gates and ready for quantitative calibration.

Chapter 12

Emergent Electroweak Sector: Structural Symmetry Breaking

Aim. Derive the $SU(2)_L \times U(1)_Y$ sector from fiber–isometries of the spectral medium, realize symmetry breaking as a *structural resonance* (a Higgs–like order parameter), and map spectral couplings to precision electroweak observables (LEP/LHC), including custodial protection, oblique parameters (S, T, U), and fermion masses/mixings.

12.1 Gauge Structure from Fiber Isometries

Let $\mathrm{Iso}(\mathcal{F}_{\mathrm{int}}) \supset SU(2)_L \times U(1)_Y$ act on left–handed doublets and hypercharge current constructed from the internal spectral fiber (Vol. I, Ch. 2–3). The spectral derivative D_μ pulls back the connection

$$\mathcal{A}_\mu \equiv g\, W_\mu^a \frac{\tau^a}{2} + g' B_\mu\, Y, \qquad F_{\mu\nu} = \partial_\mu \mathcal{A}_\nu - \partial_\nu \mathcal{A}_\mu + [\mathcal{A}_\mu, \mathcal{A}_\nu], \tag{12.1}$$

with gauge–kinetic sector on the emergent metric g_{eff}:

$$\mathcal{L}_{\mathrm{gauge}} \;=\; -\frac{1}{4}\, Z_W(\mathcal{S})\, W^a_{\mu\nu} W^{a\,\mu\nu} - \frac{1}{4}\, Z_B(\mathcal{S})\, B_{\mu\nu} B^{\mu\nu} - \frac{1}{2}\, Z_{WB}(\mathcal{S})\, W^3_{\mu\nu} B^{\mu\nu}. \tag{12.2}$$

The wavefunction factors $Z_{W,B,WB}$ are functionals of (ρ, η, χ^μ) (spectral stiffness/anisotropy). Canonical normalization fixes effective couplings $g_{\mathrm{eff}}, g'_{\mathrm{eff}}$ and a possible kinetic mixing Z_{WB} feeding into the oblique parameter S.

12.2 Structural Higgs: Order Parameter and Mass Generation

The Higgs doublet H is a *composite/collective* spectral order parameter built from the phase Φ and density/viscosity moduli (ρ, η), transforming as $(\mathbf{2}_{1/2})$ under $SU(2)_L \times U(1)_Y$. Its effective Lagrangian reads

$$\mathcal{L}_H \;=\; Z_H(\mathcal{S})\, (D_\mu H)^\dagger (D^\mu H) - V(H), \qquad V(H) = -\mu^2_{\mathrm{eff}}(\mathcal{S})\, H^\dagger H + \lambda_{\mathrm{eff}}(\mathcal{S})\, (H^\dagger H)^2 + \cdots, \tag{12.3}$$

with covariant derivative $D_\mu H = (\partial_\mu - i\frac{g}{2}\tau^a W_\mu^a - i\frac{g'}{2}B_\mu)H$. Spontaneous symmetry breaking occurs for $\mu_{\rm eff}^2 > 0$, with

$$v_{\rm eff}^2 \;=\; \frac{\mu_{\rm eff}^2}{\lambda_{\rm eff}}\,\frac{1}{Z_H}, \qquad m_h^2 \;=\; 2\,\lambda_{\rm eff}\,v_{\rm eff}^2, \tag{12.4}$$

and gauge–boson masses

$$m_W \;=\; \frac{1}{2} g_{\rm eff}\, v_{\rm eff}, \qquad m_Z \;=\; \frac{1}{2}\sqrt{g_{\rm eff}^2 + g_{\rm eff}'^2}\, v_{\rm eff}, \qquad \cos\theta_W \equiv \frac{g_{\rm eff}}{\sqrt{g_{\rm eff}^2 + g_{\rm eff}'^2}}. \tag{12.5}$$

Spectral anisotropy (χ^μ) can induce tiny direction–dependent corrections to Z_H (bounded by precision data; see Sec. 12.6).

12.3 Custodial Symmetry and the ρ Parameter

In the isotropic limit, the Higgs sector enjoys an accidental $SU(2)_{\rm cust}$ acting on (H, H^c), which enforces

$$\rho \;\equiv\; \frac{m_W^2}{m_Z^2\cos^2\theta_W} \;=\; 1 \quad\Rightarrow\quad T = 0. \tag{12.6}$$

Spectral sources that break custodial symmetry (e.g. $Z_{WB} \neq 0$, anisotropic kinetic terms, higher–dimensional operators) generate

$$\alpha\, T \;=\; \frac{\delta m_W^2}{m_W^2} - \frac{\delta m_Z^2}{m_Z^2}, \qquad \alpha\, S \;=\; 4 s_W^2 c_W^2\, \delta Z_{WB}, \qquad \alpha\, U \;=\; 4 s_W^2\,(\delta Z_W - \delta Z_Z), \tag{12.7}$$

with α the EM fine–structure constant and $s_W \equiv \sin\theta_W$. The spectral EFT ties (S, T, U) to $(Z_{W,B,WB}, Z_H$, anisotropy) with gates enforcing smallness.

12.4 Fermion Masses, Mixings, and Alignment

Fermionic excitations are topological/solitonic modes (Vol. I, Ch. 4; Vol. II, Sec. **??**), organized into $SU(2)_L$ doublets/singlets by fiber representations. Spectral Yukawas arise as overlap integrals between fermion profiles and the Higgs texture:

$$\mathcal{L}_Y \;=\; -\bar\psi_L\, Y_f(\mathcal{S})\, H\, \psi_R \;+\; \text{h.c.}, \qquad (Y_f)_{ij} \;\sim\; \int d^3x\, \mathcal{J}_{ij}[\Phi, \rho, \eta; \chi^\mu]\,, \tag{12.8}$$

yielding masses $m_{f_i} = \frac{v_{\rm eff}}{\sqrt{2}}\,(Y_f)_{ii}$ after diagonalization, and mixings from misalignment between up/down (or charged–lepton/neutrino) overlap matrices:

$$V_{\rm CKM} = U_{u_L}^\dagger U_{d_L}, \qquad U_{\rm PMNS} = U_{\ell_L}^\dagger U_{\nu_L}. \tag{12.9}$$

Hierarchies follow from exponentially sensitive overlaps (spectral localization); flavor–changing neutral currents are suppressed by *alignment* in the spectral basis (natural flavor conservation).

12.5 Unitarity in VV Scattering and Higgs–like Resonance

Longitudinal vector–boson scattering $V_L V_L \to V_L V_L$ threatens partial–wave unitarity unless a scalar resonance h (the structural Higgs) unitarizes the amplitude. Writing couplings as κ_V and κ_f relative to SM,

$$\mathcal{L}_{hVV} = \kappa_V \left(\frac{2m_W^2}{v_{\text{eff}}}\, h\, W_\mu^+ W^{-\,\mu} + \frac{m_Z^2}{v_{\text{eff}}}\, h\, Z_\mu Z^\mu \right), \qquad \mathcal{L}_{h\bar{f}f} = -\,\kappa_f\, \frac{m_f}{v_{\text{eff}}}\, h\, \bar{f} f, \tag{12.10}$$

the $J = 0$ partial wave satisfies $|a_0| < 1/2$ for $\kappa_V \simeq 1$ up to the inelastic threshold. Deviations $\delta\kappa_V$ are controlled by Z_H and higher–derivative spectral operators; LHC signal–strength fits constrain $|\delta\kappa_V| \ll 1$.

12.6 Precision Constraints and Spectral Mapping

Electroweak precision tests (LEP/SLD/Tevatron/LHC) impose

$$|S| \lesssim \mathcal{O}(10^{-1}), \quad |T| \lesssim \mathcal{O}(10^{-1}), \quad |\delta g_{Zff}|/g_{Zff} \lesssim 10^{-3}, \quad |\delta m_W| \lesssim \text{few} \times 10^{-2}\ \text{GeV}. \tag{12.11}$$

In our EFT, these map to

$$S \sim 4 s_W c_W\, Z_{WB}, \quad T \sim \frac{1}{\alpha} \left(\frac{\Delta Z_H^{(\parallel)} - \Delta Z_H^{(\perp)}}{Z_H} \right), \quad \kappa_V - 1 \sim -\tfrac{1}{2}\, \Delta Z_H,$$

with ΔZ_H the (angle–averaged or anisotropic) renormalization of the Higgs kinetic term from spectral operators. Additional vector resonances (spectral W', Z') arise as higher fiber modes with masses $M_{V'} \gtrsim \Lambda_{\text{spec}}$; direct LHC bounds $M_{V'} \gtrsim \mathcal{O}(\text{few TeV})$ constrain the spectral gap.

12.7 Gates: S/G/TQ/D for the Electroweak Sector

- **(S) Causality/hyperbolicity.** Propagation cones of (W, B, h) lie inside that of g_{eff}; $c_T^2, c_s^2 \leq 1$. Anisotropic corrections to Z's are bounded by cosmology and collider time–of–flight.
- **(G) Gauge/BRST consistency.** Gauge fixing and (emergent) BRST/FP structure ensure gauge–independent observables; anomalies cancel (see Ch. 10).
- **(TQ) Unitarity/positivity.** Forward–limit positivity on four–boson operators; VV scattering unitarized by h with κ_V near 1.
- **(D) Dissipation.** GKSL kernels, if relevant at finite density/temperature, are gauge–covariant and retarded; no superluminal signalling; widths $\Gamma \geq 0$.

12.8 Phenomenology Program and LEP/LHC Interfaces

1. **Global EW fit.** Sample spectral parameter set

$$\{Z_W, Z_B, Z_{WB}, Z_H, \mu_{\text{eff}}, \lambda_{\text{eff}};\ \epsilon_{\text{aniso}}\}$$

   and map to $\{m_W, m_Z, \sin^2\theta_W, m_h, S, T, U, \kappa_V, \kappa_f\}$; confront LEP–SLD–LHC data.

2. **Higgs portal to the medium.** Predict correlations between κ_V, κ_f and oblique parameters from common Z_H origin; test via HL–LHC/ILC/CLIC precision.

3. **Vector–boson scattering.** Compute $pp \to jj\, V_L V_L$ distributions with spectral operators; check unitarity sum rules and custodial protection.

4. **Flavor and neutrinos.** Extract CKM/PMNS from spectral overlaps; test MFV–like suppression of FCNC and lepton–flavor violation.

Outcome. The electroweak sector emerges from fiber isometries; symmetry breaking is a structural resonance that endows W, Z and fermions with masses through a calculable v_{eff} and spectral Yukawas. Precision constraints map to a small set of wavefunction/kinetic renormalizations and higher–derivative operators, with custodial symmetry ensuring $\rho \simeq 1$ and a Higgs–like scalar unitarizing VV scattering. This provides a direct bridge to LEP/LHC tests while preserving S/G/TQ/D gates.

Chapter 13

Flavor and Neutrinos: Spectral Origin of Generations

This chapter develops a concrete, testable mechanism whereby *three fermion generations*, hierarchical masses, flavor mixing, and CP violation arise from the spectral structure of the Quantum–Aether medium. Fermionic excitations are realized as normal modes (or solitonic bound states) of a *spectral Dirac operator* on an internal fiber, coupled to an emergent Higgs-like structural resonance. Yukawa textures become *overlap integrals* of internal wavefunctions with structural profiles, naturally generating hierarchies and mixings. Majorana masses originate from high-scale topological stiffness, yielding a spectral see–saw for neutrinos. We close with quantitative predictions and consistency gates.

13.1 Spectral Origin of Generations

13.1.1 Internal fiber and the spectral Dirac operator

Let the aether fields be $\mathcal{S}(x) = \{\Phi, \rho, \eta, \chi^\mu, T\}$ (Vol. I), with effective metric $g_{\text{eff}}[\mathcal{S}]$ and spectral derivative D_μ. Fermionic excitations live on a bundle $\mathcal{M} \times \mathcal{F}$, where $\mathcal{M}$ is the emergent spacetime and $\mathcal{F}$ an internal fiber with metric $\gamma_{ab}(y; \mathcal{S})$. The *spectral Dirac operator* is

$$\mathcal{D}_{\text{spec}} \;=\; i\,\Gamma^A E_A{}^\mu(\mathcal{S})\, D_\mu \;+\; i\,\Gamma^{\hat{a}} E_{\hat{a}}{}^b(\mathcal{S})\, \nabla_b \;+\; \mathcal{M}_{\text{int}}[\mathcal{S}]\,, \tag{13.1}$$

where ∇_b is the covariant derivative on $(\mathcal{F}, \gamma)$ including the internal connection $\mathcal{A}_b$ associated with fiber isometries (the seed of gauge symmetry, Vol. II). The matrices Γ^A split into spacetime and internal blocks; $\mathcal{M}_{\text{int}}[\mathcal{S}]$ encodes structural couplings (including the emergent "Higgs" profile $H(y)$).

We expand a 4D spinor field $\Psi(x, y)$ in internal modes:

$$\Psi(x, y) \;=\; \sum_n \psi_n(x)\, u_n(y)\,, \qquad \mathcal{D}_{\text{int}}\, u_n(y) \;=\; \lambda_n\, u_n(y), \tag{13.2}$$

where $\mathcal{D}_{\text{int}} \equiv i\,\Gamma^{\hat{a}} E_{\hat{a}}{}^b \nabla_b + \mathcal{M}_{\text{int}}[\,\cdot\,]$ acts purely on $\mathcal{F}$. After integrating over y, the 4D effective action contains kinetic terms for $\psi_n(x)$ and mass/Yukawa matrices determined by λ_n and overlap integrals with H.

13.1.2 Bound-state counting: why three?

Generations correspond to *normalizable, discrete* internal modes u_n lying below the internal continuum threshold λ_{cont}. Two robust routes can produce exactly three light modes:

(i) Spectral confinement with triple-well/finite-size quantization. In a WKB approximation along a preferred internal coordinate s aligned to $\hat{\chi}$,

$$N_{\text{bound}} \;\approx\; \frac{1}{\pi}\int_{s_1}^{s_2} ds\,\sqrt{U_{\text{spec}}(s)-\lambda^2}\,, \tag{13.3}$$

with U_{spec} an effective confining potential induced by (ρ,η,H) and connection $\mathcal{A}_s$. For a wide window of structural parameters, $N_{\text{bound}}=3$ occurs naturally before the first continuum band opens.

(ii) Topological index for near-zero modes. If $\mathcal{F}$ supports a nontrivial winding/Hopf class $\mathfrak{Q}$ in the background (Vol. I, Ch. 4), the spectral asymmetry of $\mathcal{D}_{\text{int}}$ can protect a finite set of near-zero modes. A *counting lemma* reads schematically

$$N_{\text{zero-like}} \;=\; \text{Index}(\mathcal{D}_{\text{int}}) \;=\; \mathfrak{Q}\,, \qquad \mathfrak{Q}=3 \;\Rightarrow\; N\simeq 3, \tag{13.4}$$

up to small lifting by $H(y)$. Either mechanism yields *three generations* as the three lowest normalizable modes $u_{0,1,2}$.

13.2 Hierarchies and Mixing from Overlaps

13.2.1 Yukawa matrices as overlap integrals

Let $H(y)$ denote the structural Higgs profile and $\mathcal{Y}_f[\mathcal{S}(y)]$ the species-dependent coupling kernel for fermion type $f\in\{u,d,\ell,\nu\}$. The 4D Yukawa matrices are

$$\big[Y_f\big]_{mn} \;=\; \int_{\mathcal{F}} d^d y\,\sqrt{\gamma(y)}\;u^{\dagger}_{m,L}(y)\;\mathcal{Y}_f[\mathcal{S}(y)]\,H(y)\;u_{n,R}(y). \tag{13.5}$$

Hierarchies arise from *geometric misalignment* of the localized modes $u_{n,L/R}$ and the profile H. If u_n are quasi-Gaussian, centered at s_n along $\hat{\chi}$ with width σ,

$$\Big|[Y_f]_{mn}\Big| \;\sim\; y_{f,0}\,\exp\!\left[-\frac{(s_{L,m}-s_{R,n})^2}{2\sigma^2}\right], \quad \epsilon \;\equiv\; \exp\!\left[-\frac{\Delta s^2}{2\sigma^2}\right]\in(0,1), \tag{13.6}$$

inducing a texture Y_f with ϵ-suppressed off-diagonals and a hierarchical spectrum.

13.2.2 Wolfenstein-like textures and CKM

Diagonalize $Y_u = U_{u_L}\,\hat{y}_u\,U^{\dagger}_{u_R}$, $Y_d = U_{d_L}\,\hat{y}_d\,U^{\dagger}_{d_R}$. The quark mixing matrix is

$$V_{\text{CKM}} = U^{\dagger}_{u_L}U_{d_L} \;\approx\; \begin{pmatrix} 1-\frac{1}{2}\lambda^2 & \lambda & A\lambda^3(\rho-i\eta_{\text{CP}}) \\ -\lambda & 1-\frac{1}{2}\lambda^2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta_{\text{CP}}) & -A\lambda^2 & 1 \end{pmatrix} + \mathcal{O}(\lambda^4), \tag{13.7}$$

with the Wolfenstein parameter identified as $\lambda\sim\epsilon$ from (13.6). The smallness of $|V_{ub}|\sim\lambda^3$ follows from triple misalignment along $\hat{\chi}$.

13.2.3 Leptons and PMNS structure

For charged leptons and neutrinos, $Y_\ell = U_{\ell_L}\hat{y}_\ell U_{\ell_R}^\dagger$, $Y_\nu = U_{\nu_L}\hat{y}_\nu U_{\nu_R}^\dagger$, the mixing is $U_{\rm PMNS} = U_{\ell_L}^\dagger U_{\nu_L}$. If neutrino internal modes are more delocalized (larger σ_ν), overlaps are less hierarchical and large angles result:

$$U_{\rm PMNS} \sim \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_{\rm CP}} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta_{\rm CP}} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta_{\rm CP}} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta_{\rm CP}} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta_{\rm CP}} & c_{23}c_{13} \end{pmatrix}, \tag{13.8}$$

with $s_{12}, s_{23} = \mathcal{O}(1/\sqrt{2})$ and $s_{13} \sim \mathcal{O}(0.1)$ emerging from moderately misaligned internal profiles.

13.3 CP Structure from Geometric Phases and WZW/CS Terms

13.3.1 Berry phases on the fiber

Let internal profiles carry phases from the connection $\mathcal{A}_b$. A loop $\mathcal{C} \subset \mathcal{F}$ yields a Berry holonomy

$$\Theta_f \;=\; \oint_{\mathcal{C}} \mathrm{Tr}\, \mathcal{A}_f \;=\; \int_\Sigma \mathrm{Tr}\, \mathcal{F}_f\,, \qquad \mathcal{F}_f = d\mathcal{A}_f + \mathcal{A}_f \wedge \mathcal{A}_f, \tag{13.9}$$

which feeds complex phases into the overlaps (13.5). Nontrivial Θ_f generates physical CP phases after rephasings.

13.3.2 Jarlskog invariant and emergent CP

The CKM CP measure

$$J_{\rm CKM} \;=\; \mathrm{Im}\, \det\Big[\, Y_u Y_u^\dagger,\; Y_d Y_d^\dagger \,\Big] \;\propto\; \epsilon^6 \sin\delta_{\rm CKM}, \tag{13.10}$$

is sourced by geometric phases and possible effective Wess–Zumino–Witten/Chern–Simons (WZW/CS) terms in the internal action, consistent with anomaly constraints (see Sec. 13.6).

13.4 Spectral See–Saw for Neutrino Masses

13.4.1 Construction

Introduce Dirac masses from overlaps and a right-handed Majorana mass from high-scale topological stiffness:

$$\mathcal{L}_\nu \supset -\,\bar{L}\, Y_\nu H\, \nu_R \;-\; \tfrac{1}{2}\, \nu_R^\top C\, M_R\, \nu_R \;+\; \text{h.c.} \tag{13.11}$$

with $m_D = v_{\rm eff} Y_\nu/\sqrt{2}$ and $M_R \sim \Lambda_{\rm spec}\, e^{-S_{\rm top}}$ set by a spectral/topological barrier. Integrating out ν_R gives

$$m_\nu \;=\; -\, m_D\, M_R^{-1}\, m_D^\top, \qquad m_{\nu,i} \sim \frac{m_{D,i}^2}{M_R}\,. \tag{13.12}$$

A modest hierarchy in m_D produces normal ordering and observed splittings Δm^2.

13.4.2 Neutrinoless double beta and phases

The effective mass in $0\nu\beta\beta$ reads

$$m_{\beta\beta} = \left| \sum_i (U_{\mathrm{PMNS}})_{ei}^2 \, m_{\nu,i} \right|, \tag{13.13}$$

with Majorana phases determined by internal geometric phases. The framework predicts a band for $m_{\beta\beta}$ tied to $\sum m_\nu$ and the delocalization scale of $u_{\nu,i}$.

13.5 Predictions and Constraints

Mass hierarchies (quarks). From (13.6), $m_u : m_c : m_t \sim \epsilon^{2a_u} : \epsilon^{a_u} : 1$, and similarly for down-type; fit yields $\epsilon \simeq \lambda \simeq 0.22$.

CKM pattern. $|V_{us}| \sim \epsilon$, $|V_{cb}| \sim \epsilon^2$, $|V_{ub}| \sim \epsilon^3$, with a CP phase from (13.9) of the observed order.

PMNS. Large θ_{12}, θ_{23} and moderate θ_{13} from broader neutrino profiles; δ_{CP} linked to internal phases, potentially near $-\pi/2$.

Cosmology. $\sum m_\nu$ bounded by the internal length scales; the model maps $\sum m_\nu \leftrightarrow (\sigma_\nu, \Lambda_{\mathrm{spec}})$.

Leptogenesis window. If ν_R decays inherit CP phases from internal geometry, standard thermal leptogenesis is viable for $M_R \gtrsim 10^9$ GeV (subject to reheating bounds of Vol. II).

13.6 Consistency Gates and Anomaly Matching

- **(S) Stability & hyperbolicity:** fermion kinetic forms positive; front velocities within the g_{eff} cone.
- **(G) Gauge emergence & BRST:** isometries of $\mathcal{F}$ generate the effective gauge group; Ward identities hold in the overlap construction (Vol. IV).
- **(TQ) Anomalies:** ABJ and mixed anomalies cancel in the 4D effective theory; WZW/CS terms consistent with 't Hooft matching.
- **(D) Data compliance:** CKM/PMNS angles and CP phases within experimental bounds; $\sum m_\nu$ and $m_{\beta\beta}$ compatible with cosmology and $0\nu\beta\beta$ limits.

13.7 Numerical Protocol and Parameter Map

Protocol. Choose $\gamma_{ab}(y)$, $\mathcal{A}_b(y)$, $H(y)$ from structural backgrounds $\mathcal{S}$. Solve $\mathcal{D}_{\mathrm{int}} u_n = \lambda_n u_n$ (finite elements / spectral collocation on $\mathcal{F}$). Assemble Y_f via (13.5); compute $V_{\mathrm{CKM}}, U_{\mathrm{PMNS}}$. Scan small-$\epsilon$ textures by shifting centers s_n along $\hat{\chi}$.

Parameter map.

$$\Big(\ \sigma,\ \Delta s,\ \Theta_f,\ \Lambda_{\rm spec},\ v_{\rm eff}\ \Big)\ \longmapsto\ \Big(\ Y_f,\ \{m_f\},\ V_{\rm CKM},\ U_{\rm PMNS},\ m_\nu,\ m_{\beta\beta}\ \Big).$$

The overlap scale $\epsilon = \exp[-(\Delta s)^2/(2\sigma^2)]$ controls quark hierarchies and CKM; delocalization σ_ν controls PMNS; Θ_f fixes CP phases; $\Lambda_{\rm spec}$ sets M_R and $\sum m_\nu$.

Summary: In the spectral medium, **three** generations emerge as the *three lowest* internal fermion modes; mass hierarchies and mixings are *geometric overlaps*; CP violation is *geometric/topological*; light neutrino masses follow from a *spectral see–saw*. The construction is gated by stability, gauge, anomaly, and data constraints, yielding a compact, predictive bridge from spectral structure to the observed flavor sector.

Chapter 14

Thermodynamics and Hydrodynamics of the Spectral Medium

We develop the macroscopic (hydro/thermo) description of the Quantum–Aether medium, deriving local conservation laws, constitutive relations with anisotropic transport, entropy production and fluctuation–dissipation, linear modes (sound/shear), and Kubo formulae. All constructions are explicitly tied to the dissipation gate (D) and to hyperbolicity/causality requirements set by the emergent metric $g_{\mathrm{eff}}[\mathcal{S}]$ (Vol. I, II).

14.1 Local Equilibrium, State Variables, and Balances

Aether frame and projectors. Let u^μ be the aether rest frame (unit timelike in g_{eff}). Define the spatial projector $\Delta^{\mu\nu} \equiv g_{\mathrm{eff}}^{\mu\nu} + u^\mu u^\nu$. The internal director $\hat{\chi}^\mu$ (unit spacelike, $\hat{\chi}^\mu u_\mu = 0$) induces uniaxial projectors

$$P_\parallel^{\mu\nu} \equiv \hat{\chi}^\mu \hat{\chi}^\nu, \qquad P_\perp^{\mu\nu} \equiv \Delta^{\mu\nu} - \hat{\chi}^\mu \hat{\chi}^\nu. \tag{14.1}$$

Thermodynamics. Local state is specified by $(\varepsilon, s, n;\, \rho, \eta, \dots)$ with energy density ε, entropy density s, and a conserved scalar density n (e.g. a structural "charge"); temperature ϑ and chemical potential μ follow from

$$d\varepsilon = \vartheta\, ds + \mu\, dn\,, \qquad p = -\varepsilon + \vartheta s + \mu n \quad \text{(Gibbs–Duhem)}. \tag{14.2}$$

Conservation laws. Let $T^{\mu\nu}$ be the total stress–energy, J^μ a conserved current.

$$\nabla_\mu T^{\mu\nu} = 0, \qquad \nabla_\mu J^\mu = 0, \tag{14.3}$$

where ∇ is the Levi–Civita connection of g_{eff}. Decompose in the Landau–aether frame:

$$T^{\mu\nu} = \varepsilon u^\mu u^\nu + (p + \Pi)\Delta^{\mu\nu} + u^{(\mu} q^{\nu)} + \pi^{\mu\nu}, \qquad J^\mu = n u^\mu + \nu^\mu, \tag{14.4}$$

with bulk correction Π, heat/energy flux $q^\mu u_\mu = 0$, diffusion $\nu^\mu u_\mu = 0$, and traceless shear stress $\pi^{\mu\nu} u_\nu = 0$, $\pi^\mu{}_\mu = 0$.

14.2 Constitutive Relations with Uniaxial Anisotropy

Kinematics. Define expansion, acceleration, vorticity, and shear

$$\theta \equiv \nabla_\mu u^\mu, \qquad a^\mu \equiv u^\nu \nabla_\nu u^\mu, \qquad \omega^{\mu\nu} \equiv \tfrac{1}{2}\Delta^{\mu\alpha}\Delta^{\nu\beta}(\nabla_\alpha u_\beta - \nabla_\beta u_\alpha),$$
$$\sigma^{\mu\nu} \equiv \Delta^{\mu\alpha}\Delta^{\nu\beta}\Big(\nabla_{(\alpha} u_{\beta)} - \tfrac{1}{3}\theta\,\Delta_{\alpha\beta}\Big). \tag{14.5}$$

First-order (Navier–Stokes) sector. Allow distinct transport coefficients parallel/perpendicular to $\hat{\chi}$:

$$\Pi = -\,\zeta\,\theta, \tag{14.6}$$
$$q^\mu = -\,\kappa_\perp\, P_\perp^{\mu\nu}\,\nabla_\nu\vartheta \;-\; \kappa_\parallel\, P_\parallel^{\mu\nu}\,\nabla_\nu\vartheta \;-\; \alpha_\times\,\epsilon^\mu{}_{\alpha\beta\gamma}\,u^\alpha \hat{\chi}^\beta \nabla^\gamma\vartheta, \tag{14.7}$$
$$\pi^{\mu\nu} = -2\eta_\perp\,\sigma_\perp^{\mu\nu} - 2\eta_\parallel\,\sigma_\parallel^{\mu\nu} - 2\eta_\times\,\Sigma^{\mu\nu}, \tag{14.8}$$

with projections

$$\sigma_\parallel^{\mu\nu} \equiv \Big(P_\parallel^{\mu\alpha}P_\parallel^{\nu\beta} - \tfrac{1}{3}P_\parallel^{\mu\nu}P_{\parallel\,\alpha\beta}\Big)\sigma_{\alpha\beta}, \qquad \sigma_\perp^{\mu\nu} \equiv \Big(P_\perp^{\mu\alpha}P_\perp^{\nu\beta} - \tfrac{1}{2}P_\perp^{\mu\nu}P_{\perp\,\alpha\beta}\Big)\sigma_{\alpha\beta},$$
$$\Sigma^{\mu\nu} \equiv P_\parallel^{(\mu|\alpha|}P_\perp^{\nu)\beta}\,\sigma_{\alpha\beta}, \tag{14.9}$$

and $\epsilon^{\mu\nu\alpha\beta}$ the g_{eff}-Levi–Civita tensor. Reciprocity (Onsager) requires the cross-coefficients (e.g. $\alpha_\times$, $\eta_\times$) to satisfy symmetry constraints when time-reversal holds.

Positivity (Gate D). The quadratic entropy production (Sec. 14.3) imposes

$$\kappa_{\perp,\parallel} \geq 0, \quad \zeta \geq 0, \quad \eta_{\perp,\parallel} \geq 0, \quad \text{and} \quad \begin{pmatrix}\kappa_\perp & \cdots\\ \cdots & \eta_\perp\end{pmatrix}, \begin{pmatrix}\kappa_\parallel & \cdots\\ \cdots & \eta_\parallel\end{pmatrix} \text{ positive semidefinite.} \tag{14.10}$$

Second order (Israel–Stewart) for causality. To restore hyperbolicity, promote dissipative fluxes to dynamical fields with relaxation:

$$\begin{aligned}
\tau_\Pi\, u^\alpha\nabla_\alpha\Pi + \Pi &= -\,\zeta\,\theta + \cdots,\\
\tau_q\,\Delta^\mu{}_\nu\, u^\alpha\nabla_\alpha q^\nu + q^\mu &= -\,\kappa_\perp P_\perp^{\mu\nu}\nabla_\nu\vartheta - \kappa_\parallel P_\parallel^{\mu\nu}\nabla_\nu\vartheta + \cdots,\\
\tau_{\pi,\perp}\,\Delta^{\mu\nu}{}_{\alpha\beta}u^\lambda\nabla_\lambda\pi_\perp^{\alpha\beta} + \pi_\perp^{\mu\nu} &= -2\eta_\perp\,\sigma_\perp^{\mu\nu} + \cdots,\\
\tau_{\pi,\parallel}\,\Delta^{\mu\nu}{}_{\alpha\beta}u^\lambda\nabla_\lambda\pi_\parallel^{\alpha\beta} + \pi_\parallel^{\mu\nu} &= -2\eta_\parallel\,\sigma_\parallel^{\mu\nu} + \cdots,
\end{aligned} \tag{14.11}$$

with $\Delta^{\mu\nu}{}_{\alpha\beta}$ the symmetric traceless projector. Positive τ's ensure causal signal speeds and well-posedness.

14.3 Entropy Current and the Dissipation Gate

Define the canonical entropy current (Landau frame)

$$S^\mu \equiv s\,u^\mu - \frac{1}{\vartheta}\,q^\mu - \frac{\mu}{\vartheta}\,\nu^\mu, \tag{14.12}$$

and use (14.3) together with (14.6)–(14.8) to obtain

$$\nabla_\mu S^\mu = \frac{\zeta}{\vartheta}\,\theta^2 + \frac{1}{\vartheta}\,\pi^{\mu\nu}\sigma_{\mu\nu} + \kappa_\perp\, P_\perp^{\mu\nu}\,\frac{\nabla_\mu\vartheta\,\nabla_\nu\vartheta}{\vartheta^2} + \kappa_\parallel\,(\hat\chi\cdot\nabla\vartheta)^2\,\frac{1}{\vartheta^2} + \cdots \;\geq 0. \tag{14.13}$$

Thus Gate (D) is satisfied provided the transport matrices are positive semidefinite (14.10). In the spectral model, $\eta > 0$ ensures a microscopic origin for the macroscopic irreversibility (Vol. II, Sec. η-wave).

14.4 Linear Modes: Sound, Shear, and Anisotropy

Linearize about uniform equilibrium and consider plane waves $\propto e^{-i\omega t + i\mathbf{k}\cdot\mathbf{x}}$.

Parallel propagation ($\mathbf{k} \parallel \hat\chi$). Longitudinal (sound) modes:

$$\omega_\parallel^\pm \;=\; \pm c_{s,\parallel}\,k \;-\; i\,\Gamma_{s,\parallel}\,k^2 + \mathcal{O}(k^3), \qquad \Gamma_{s,\parallel} = \frac{1}{2(\varepsilon+p)}\Big(\tfrac{4}{3}\eta_\parallel + \zeta\Big), \tag{14.14}$$

Shear diffusion:

$$\omega_{\mathrm{sh},\parallel} \;=\; -\,i\,\nu_\parallel\,k^2 + \cdots, \qquad \nu_\parallel \equiv \frac{\eta_\parallel}{\varepsilon+p}. \tag{14.15}$$

Perpendicular propagation ($\mathbf{k} \perp \hat\chi$). Longitudinal:

$$\omega_\perp^\pm \;=\; \pm c_{s,\perp}\,k \;-\; i\,\Gamma_{s,\perp}\,k^2 + \cdots, \qquad \Gamma_{s,\perp} = \frac{1}{2(\varepsilon+p)}\Big(\tfrac{4}{3}\eta_\perp + \zeta\Big), \tag{14.16}$$

Shear (one transverse polarization propagates in the $\hat\chi$–plane):

$$\omega_{\mathrm{sh},\perp} \;=\; -\,i\,\nu_\perp\,k^2 + \cdots, \qquad \nu_\perp \equiv \frac{\eta_\perp}{\varepsilon+p}. \tag{14.17}$$

CMB/LSS isotropy bounds imply $|c_{s,\parallel} - c_{s,\perp}|/c_s \ll 1$ at late times, feeding direct constraints on $\eta_\parallel - \eta_\perp$.

Causality/hyperbolicity. With (14.11), all signal front velocities remain within the light cone of g_{eff}: $v_{\mathrm{front}} \leq c_{\mathrm{eff}}$ (Gate G/S).

14.5 Linear Response, Kubo Formulae, and FDT

Define retarded Green functions $G^R_{AB}(\omega,\mathbf{k}) = -i\int d^4x\, e^{i\omega t - i\mathbf{k}\mathbf{x}}\,\Theta(t)\langle[A(x),B(0)]\rangle$. With $\hat z \parallel \hat\chi$:

Shear viscosities.

$$\begin{aligned}\eta_\perp &= \lim_{\omega\to0}\frac{1}{\omega}\,\mathrm{Im}\,G^R_{T^{xy}T^{xy}}(\omega,\mathbf{k}{=}0),\\ \eta_\parallel &= \lim_{\omega\to0}\frac{1}{\omega}\,\mathrm{Im}\,G^R_{T^{xz}T^{xz}}(\omega,\mathbf{k}{=}0).\end{aligned} \tag{14.18}$$

Bulk viscosity.

$$\zeta = \frac{1}{9} \lim_{\omega\to 0} \frac{1}{\omega} \operatorname{Im} G^R_{T^{i}{}_{i}\, T^{j}{}_{j}}(\omega, 0). \tag{14.19}$$

Thermal conductivities. Let $J^i_Q \equiv T^{0i} - \frac{\varepsilon+p}{n} J^i$ be the heat current.

$$\begin{aligned} \kappa_\perp &= \frac{1}{\vartheta} \lim_{\omega\to 0} \frac{1}{\omega} \operatorname{Im} G^R_{J^x_Q J^x_Q}(\omega, 0), \\ \kappa_\parallel &= \frac{1}{\vartheta} \lim_{\omega\to 0} \frac{1}{\omega} \operatorname{Im} G^R_{J^z_Q J^z_Q}(\omega, 0). \end{aligned} \tag{14.20}$$

Fluctuation–dissipation. For each channel, $S_{AA}(\omega, 0) = \coth(\omega/2\vartheta) \operatorname{Im} G^R_{AA}(\omega, 0)$, linking noise spectra to transport, to be used in stochastic hydro simulations.

Spectral origin. In the microscopic model, transport is controlled by the spectral viscosity field $\eta(x)$ and correlators of stress–stress and current–current operators built from $\mathcal{S}$. Schematically,

$$\eta_{\perp,\parallel} \;\sim\; \int_0^\infty dt\; \langle \mathcal{T}^{xy/xz}(t)\, \mathcal{T}^{xy/xz}(0) \rangle_{\text{conn}}, \qquad \kappa_{\perp,\parallel} \;\sim\; \int_0^\infty dt\; \langle J^{x/z}_Q(t)\, J^{x/z}_Q(0) \rangle, \tag{14.21}$$

providing a parameter map from $(\rho, \eta, \hat{\chi})$ to macroscopic transport.

14.6 Numerical Protocols and Validation

Benchmark tests. (i) Shock tube (Riemann problem) with small $\eta_{\perp,\parallel}, \zeta$; (ii) Couette/Poiseuille flow to extract $\eta_{\perp,\parallel}$; (iii) Thermal diffusion to measure $\kappa_{\perp,\parallel}$; (iv) Sound attenuation to validate $\Gamma_{s,\parallel/\perp}$.

Well-posedness checks. Verify positivity (14.10), relaxation times $\tau > 0$, energy conservation to $\mathcal{O}(\Delta t^2)$, and discrete entropy production $\Delta S \geq 0$ per step. Front velocities measured from impulse tests must obey $v_{\text{front}} \leq c_{\text{eff}}$.

Data interfaces. Late-time isotropy constraints (CMB) enforce $|\eta_\parallel - \eta_\perp|/(\eta_\parallel + \eta_\perp) \ll 1$; LSS line-of-sight damping constrains κ. These bounds feed the global gate set (S/G/TQ/D).

Summary. The spectral medium admits a consistent, causal hydrodynamic closure with uniaxial transport tensors dictated by $\hat{\chi}$. Entropy production is manifestly nonnegative, transport coefficients obey Kubo/Onsager relations, and linear modes display controlled anisotropy constrained by observations. Gate (D) is thereby implemented from first principles and tied to the microscopic η-sector.

Chapter 15

Quantum Field Theory on an Effective Curved Spacetime

We formulate quantum field theory (QFT) on the *emergent* Lorentzian geometry $g_{\rm eff}[\mathcal{S}]$ generated by the spectral medium (Vol. I–II). Linearized excitations around ordered backgrounds propagate as quantum fields on $(\mathcal{M}, g_{\rm eff})$, with small, controlled deviations (e.g. weak anisotropy from $\hat{\chi}^\mu$). We develop vacuum choices (Hadamard states), particle creation in nonstationary backgrounds, detector thermality (Unruh) and horizon radiation (Hawking), and the renormalized stress–energy with semiclassical backreaction, then extend to open-system (GKSL) corrections consistent with causality and Gate (D).

15.1 Free Fields on $(\mathcal{M}, g_{\rm eff})$: Equations and Inner Product

Consider a representative scalar fluctuation φ (canonically normalized mode extracted from $\delta\mathbf{S}$) obeying

$$\Big(\Box_{g_{\rm eff}} + m_{\rm eff}^2 + \xi\, R_{\rm eff}\Big)\,\varphi \;=\; 0, \qquad \Box_{g_{\rm eff}} \equiv g_{\rm eff}^{\mu\nu}\nabla_\mu\nabla_\nu, \tag{15.1}$$

with effective mass $m_{\rm eff}$ and curvature coupling ξ determined by the quadratic spectral action (Vol. II, App. A). The conserved Klein–Gordon inner product on a Cauchy surface Σ is

$$(\varphi_1,\varphi_2) \;\equiv\; i\!\int_\Sigma d\Sigma^\mu \left(\varphi_1^*\nabla_\mu\varphi_2 - \varphi_2\nabla_\mu\varphi_1^*\right), \qquad \nabla_\mu d\Sigma^\mu = 0, \tag{15.2}$$

ensuring mode orthonormality and a consistent Fock construction. Small uniaxial anisotropies enter as $c_s^2(\theta) = 1 + \delta c_s(\theta)$ in the principal symbol; they are constrained by Gates (S/G) and will be treated perturbatively where needed.

15.2 Vacuum Choices and the Hadamard Condition

A physically admissible vacuum $|0\rangle$ must be *Hadamard*: its two-point function $G^+(x,x') = \langle 0|\varphi(x)\varphi(x')|0\rangle$ has the local short-distance form

$$G^+(x,x') \;\sim\; \frac{U(x,x')}{\sigma_+(x,x')} + V(x,x')\log\!\big[\mu^2\sigma_+(x,x')\big] + W(x,x'), \tag{15.3}$$

where σ_+ is Synge's world function with the Feynman $i\epsilon$-prescription, and U, V, W are smooth biscalars fixed by $g_{\rm eff}$ and the operator in (15.1). Hadamard-ness guarantees finite, state-independent counterterms for $T_{\mu\nu}$. On stationary spacetimes, the *ground* (lowest-energy) or *KMS thermal* states are Hadamard; on slowly varying backgrounds we adopt adiabatic vacua (Parker order $N \geq 2$).

15.3 Particle Creation in Expanding Backgrounds

For a spatially homogeneous background with line element $ds^2_{\rm eff} = -dt^2 + a^2(t)\, d\mathbf{x}^2$, expand $\varphi = \int \frac{d^3k}{(2\pi)^{3/2}} \Big[a_{\mathbf{k}} u_k(t) e^{i\mathbf{k}\cdot\mathbf{x}} + a^\dagger_{\mathbf{k}} u^*_k(t) e^{-i\mathbf{k}\cdot\mathbf{x}}\Big]$, with mode equation

$$\ddot{u}_k + \omega_k^2(t)\, u_k \;=\; 0, \qquad \omega_k^2(t) = c_s^2(t)\, \frac{k^2}{a^2(t)} + m^2_{\rm eff} + \xi R_{\rm eff} - \frac{\ddot{a}}{a}. \tag{15.4}$$

Two adiabatic regimes define *in*/*out* vacua; the Bogoliubov map yields $u_k^{\rm in} = \alpha_k u_k^{\rm out} + \beta_k u_k^{\rm out\,*}$, with particle number

$$N_k \;=\; |\beta_k|^2, \qquad |\alpha_k|^2 - |\beta_k|^2 = 1. \tag{15.5}$$

The adiabaticity condition $|\dot{\omega}_k/\omega_k^2| \ll 1$ suppresses creation; departures (e.g. near an η-driven transition, Vol. II) imprint calculable spectra and feed $\Delta N_{\rm eff}$ constraints (Vol. II, Chap. **??**).

15.4 Detector Thermality: Unruh Effect on $g_{\rm eff}$

For a uniformly accelerated detector following $x^\mu(\tau)$ with proper acceleration a (in the $g_{\rm eff}$-metric), the response rate is

$$\dot{\mathcal{F}}(E) = \int_{-\infty}^{+\infty} ds\; e^{-iEs}\, G^+\Big(x(\tau), x(\tau+s)\Big). \tag{15.6}$$

For Hadamard vacua of a massless field with $c_s \to 1$, the Wightman function along the trajectory is thermal, giving

$$\dot{\mathcal{F}}(E) \;\propto\; \frac{1}{e^{E/T_U} - 1}, \qquad T_U = \frac{a}{2\pi}. \tag{15.7}$$

Small anisotropies $\delta c_s(\theta)$ induce subleading directional corrections to the detector spectrum; Gate (S) enforces their suppression at late times.

15.5 Horizons and Hawking Radiation of the Effective Geometry

If $g_{\rm eff}$ admits a Killing horizon $\mathcal{H}$ with surface gravity κ (e.g. effective black-hole analogs, or cosmological horizons during an η-wave epoch), mode analysis near $\mathcal{H}$ yields a thermal flux at

$$T_H = \frac{\kappa}{2\pi}, \qquad \langle \mathcal{N}_\omega \rangle \;=\; \frac{\Gamma_\ell(\omega)}{e^{\omega/T_H} - 1}, \tag{15.8}$$

with greybody factors $\Gamma_\ell(\omega)$ from the effective potential of $(\Box_{g_{\rm eff}})$. Consistency requires $c_T \simeq 1$ for graviton-like sectors (multi-messenger gate, Vol. II).

15.6 Renormalized $\langle T_{\mu\nu}\rangle$ and Semiclassical Backreaction

Define the renormalized stress–energy via covariant point-splitting:

$$\langle T_{\mu\nu}\rangle_{\rm ren} = \lim_{x'\to x} \mathcal{D}_{\mu\nu}\Big[G^{+}(x,x') - H(x,x')\Big], \tag{15.9}$$

where H is the Hadamard parametrix and $\mathcal{D}_{\mu\nu}$ a state-independent bidifferential operator. The trace anomaly (for conformal fields) takes the standard form

$$\langle T^{\mu}{}_{\mu}\rangle = \frac{1}{2880\pi^2}\Big(c\,C^2_{\alpha\beta\gamma\delta} - a\,E_4 + b\,\Box R_{\rm eff}\Big), \tag{15.10}$$

with C^2 the Weyl invariant and E_4 the Euler density; coefficients depend on species. Backreaction enters the emergent dynamics as

$$\mathcal{G}_{\mu\nu}[g_{\rm eff};\mathcal{S}] = \frac{1}{M^2_{\rm eff}(\rho)}\Big(\langle T_{\mu\nu}\rangle_{\rm ren} + T^{(\rm struct)}_{\mu\nu}\Big), \tag{15.11}$$

closing the semiclassical system (cf. Vol. I, unified action). Gate (TQ): $\langle T_{\mu\nu}\rangle_{\rm ren}$ must be finite (Hadamard) and satisfy quantum energy inequalities (QEIs) to avoid pathologies.

15.7 Open-System Extensions: Covariant GKSL on Curved Backgrounds

Environmental couplings (finite $\eta > 0$, dense ρ) induce Lindblad terms covariantly:

$$\dot\rho = -i[H_{\rm eff},\rho] - \tfrac{1}{2}\int d\Sigma\,\sqrt{-g_{\rm eff}}\sum_a \gamma_a(x)\,\Big(L_a^\dagger L_a\,\rho + \rho\,L_a^\dagger L_a - 2L_a\rho L_a^\dagger\Big), \tag{15.12}$$

with local $L_a = L_a[\mathcal{S};D_\mu]$ and rates $\gamma_a(\mathcal{S}) \geq 0$. The fluctuation–dissipation relation is expressed via the noise kernel $N(x,y) = \frac{1}{2}\langle\{\hat\Xi(x),\hat\Xi(y)\}\rangle$ and the retarded self-energy Σ^R:

$$N(\omega,\mathbf{x}) = \coth\!\Big(\frac{\omega}{2\vartheta_{\rm eff}}\Big)\,\mathrm{Im}\,\Sigma^R(\omega,\mathbf{x}), \tag{15.13}$$

ensuring CP-divisibility and monotone entropy (Gate D) while preserving microcausality (the commutator vanishes outside the $g_{\rm eff}$ light cone).

15.8 Gates and Empirical Interfaces

- **(S/G)** Microcausality: the Pauli–Jordan commutator has support on the $g_{\rm eff}$ light cone; front velocities $v_{\rm front} \leq c_{\rm eff}$.
- **(TQ)** Hadamard state, finite $\langle T_{\mu\nu}\rangle_{\rm ren}$, QEIs respected.
- **(D)** Open-system terms satisfy CP and FDT; $\dot S_{\rm vN} \geq 0$.
- **Imprints.** Cosmological particle creation $\Rightarrow$ bounds from $\Delta N_{\rm eff}$; horizon thermality $\Rightarrow$ analog Hawking in laboratory media; Unruh response $\Rightarrow$ accelerated-detector tests in optical/atomic analogs.

Summary. Quantization on the emergent geometry g_{eff} reproduces the standard toolbox of QFT in curved spacetime—vacua, particle creation, Unruh/Hawking, and renormalized backreaction—while admitting controlled, testable departures (weak anisotropy, open dynamics) rooted in the spectral medium. This unifies "quantum on curved" within the same pre-geometric framework developed in Vol. I–III.

Chapter 16

Compact Objects, Horizons, Wormholes, and Causality

We analyze stationary and strong–field solutions of the emergent geometry $g_{\rm eff}[\mathcal{S}]$ sourced by the spectral medium $\mathcal{S}(x) = \{\Phi, \rho, \eta, \chi^\mu, T\}$. Our goals are: (i) formulate compact configurations and horizon conditions; (ii) assess ultracompact, horizonless *spectral solitons* as black–hole mimickers; (iii) state the status of wormholes and chronology protection under Gates (S/G/D). The effective field equations follow from the unified action (Vol. I), schematically

$$\mathcal{G}_{\mu\nu}[g_{\rm eff};\mathcal{S}] \;=\; \frac{1}{M_{\rm eff}^2(\rho)}\Big(T_{\mu\nu}^{(\rm struct)} + T_{\mu\nu}^{(\rm qu)}\Big), \qquad \nabla_\mu^{(g_{\rm eff})} T_{\rm tot}^{\mu\nu} = 0, \tag{16.1}$$

where $T_{\mu\nu}^{(\rm struct)}$ encodes the medium (including anisotropy from $\hat{\chi}^\mu$ and viscous pieces $\propto \eta$) and $T_{\mu\nu}^{(\rm qu)}$ accounts for renormalized quantum and open–system corrections (Vol. III, Chap. 15).

16.1 Stationary Spherical Configurations and Spectral TOV

Assuming a static, spherically symmetric ansatz,

$$ds_{\rm eff}^2 = -e^{2\Phi(r)} f(r)\, dt^2 + \frac{dr^2}{f(r)} + r^2 d\Omega^2, \qquad f(r) = 1 - \frac{2\,m(r)}{r}, \tag{16.2}$$

with a comoving aether frame $u^\mu = e^{-\Phi} f^{-1/2}\, \delta^\mu{}_t$. The medium supports *radial* and *tangential* pressures p_r, p_t and density ϵ, forming an anisotropic but causal stress tensor,

$$T^\mu{}_\nu = \mathrm{diag}(-\epsilon,\, p_r,\, p_t,\, p_t) \;+\; \Pi^\mu{}_\nu[\eta, \hat{\chi}], \quad \Pi^\mu{}_\nu u^\nu = 0, \quad \Pi^\mu{}_\mu = 0, \tag{16.3}$$

where Π contains shear/viscous terms that vanish in the static limit (Gate D). Einstein–like equations give

$$m'(r) = 4\pi r^2 \frac{\epsilon}{M_{\rm eff}^2}, \qquad \Phi'(r) = \frac{m + 4\pi r^3 p_r / M_{\rm eff}^2}{r\big(r - 2m\big)} \;+\; \delta\Phi_\chi, \tag{16.4}$$

with $\delta\Phi_\chi$ a bounded anisotropy correction $\propto (\hat{\chi}\cdot\hat{r})^2$. Conservation $\nabla_\mu T^{\mu r} = 0$ yields the *spectral TOV* equation

$$\frac{dp_r}{dr} = -(\epsilon + p_r)\,\Phi' \;+\; \frac{2}{r}\,\Delta \;+\; \mathcal{O}(\partial_r M_{\rm eff}), \qquad \Delta \equiv p_t - p_r, \tag{16.5}$$

with causal bound $|\Delta| \lesssim \epsilon$ from hyperbolicity (Gate G). Regular centers require $m \sim (4\pi/3)\epsilon_c r^3/M_{\rm eff}^2$ and finite $\Phi(0)$. Given an EoS from the spectral energy density $\mathcal{E}[\mathcal{S}]$ (Vol. I), Eqs. (16.4)–(16.5) determine radius R and gravitational mass $M = m(R)$.

Compactness and Buchdahl-type bounds. For isotropic, monotone profiles one recovers $\frac{2M}{R} \le \frac{8}{9}$ (Buchdahl). With controlled anisotropy $\Delta \ge 0$ one can approach the photon–sphere threshold $R \to 3M$ without forming a horizon (cf. Sec. 16.3). Gate (S) enforces $c_T^2, c_s^2 \le 1$, disfavoring super-extreme compactness.

16.2 Horizons, Surface Gravity, and Entropy Balance

A Killing horizon occurs when $f(r_h) = 0$ for the stationary Killing field $\xi^\mu = (\partial_t)^\mu$. Surface gravity is

$$\kappa \;=\; \frac{1}{2}\, e^{\Phi(r_h)}\, f'(r_h). \tag{16.6}$$

Raychaudhuri's equation for null congruences k^μ on $\mathcal{H}$ gives the area law $dA/d\lambda \ge 0$ if the effective null energy condition (NEC) $T^{\rm tot}_{\mu\nu}k^\mu k^\nu \ge 0$ holds. The generalized entropy

$$S_{\rm gen} \;=\; \frac{\mathcal{A}}{4\,G_{\rm eff}} \;+\; S_{\rm out} \qquad \left(G_{\rm eff} \equiv [M_{\rm eff}^2]^{-1}\right) \tag{16.7}$$

is non-decreasing provided the open–system production $\dot{S}_{\rm out} \ge 0$ (Gate D) and $G_{\rm eff}$ varies adiabatically. Viscous boundary terms at $\mathcal{H}$ define a membrane paradigm with shear coefficient $\eta_{\rm H} = \frac{M_{\rm eff}^2}{16\pi}$ up to spectral corrections; positivity follows from Gate (TQ).

16.3 Ultracompact, Horizonless Spectral Solitons

The structural Lagrangian (Vol. I) admits *localized, nontopological* condensates whose pressure support includes spectral stiffness and quartic (Skyrme-like) terms. They realize static solutions with $R > 2M$ but potentially $R < 3M$, thereby possessing a photon ring without an event horizon.

Existence and stability. Let $\mathcal{E}[\mathcal{S}] = \epsilon(\rho, \Phi, \dots)$ and define $\Gamma \equiv d\ln p_r / d\ln\epsilon$. Linear radial stability requires

$$\frac{dM}{d\epsilon_c} > 0, \qquad 0 < c_{s,r}^2 \equiv \frac{\partial p_r}{\partial \epsilon} \le 1, \qquad |\Delta| \le \alpha\,\epsilon, \quad 0 \le \alpha < 1, \tag{16.8}$$

with a discrete spectrum of normal modes $\omega_n^2 > 0$. Anisotropy from $\hat{\chi}^\mu$ enters as bounded shifts in the effective elastic moduli.

Observables: echoes, Love numbers, shadows. For a reflectivity $\mathcal{R}$ at $r = R$ (e.g. phase–stiff surface), post–merger GW ringdown displays *echoes* with delay

$$\Delta t_{\rm echo} \;\simeq\; 2\int_{r_*(R)}^{r_*(r_{\rm peak})} dr_* \;\approx\; 2M\,\ln\frac{1}{\epsilon_{\rm gap}}, \qquad \epsilon_{\rm gap} \equiv \frac{R-2M}{2M}. \tag{16.9}$$

Tidal Love numbers k_2 are generically nonzero (contrast with BHs), offering a strong discriminant with LIGO/Virgo/KAGRA/PTA data. EHT shadows retain photon-ring features; sub-horizon reflectivity modifies subrings marginally.

16.4 Wormholes: Throats, Energy Conditions, and QEIs

Adopt the Morris–Thorne form

$$ds^2_{\rm eff} = -e^{2\Phi_w(r)}dt^2 + \frac{dr^2}{1-b(r)/r} + r^2 d\Omega^2, \tag{16.10}$$

with a throat at r_0 where $b(r_0) = r_0$ and flare-out $b'(r_0) < 1$. Along a radial null vector k^μ, the (effective) ANEC reads

$$\int_\gamma T^{\rm tot}_{\mu\nu} k^\mu k^\nu \, d\lambda \;<\; 0 \quad \text{(traversability)}. \tag{16.11}$$

Decompose $T^{\rm tot} = T^{(\rm struct)} + \langle T\rangle_{\rm ren} + T^{(\rm open)}$.

- *Structural sector:* Spectral stresses can provide negative radial pressure, but Gates (S/G) forbid ghostly or superluminal support; hence sustained macroscopic ANEC violation from $T^{(\rm struct)}$ is strongly constrained.
- *Quantum sector:* Negative energy is bounded by quantum energy inequalities (QEIs), e.g. for an inertial sampling of width τ_0, $\int d\tau\, \rho(\tau)\, f^2(\tau) \geq -C/\tau_0^4$, with C set by field content.
- *Open sector:* GKSL noise/dissipation yields $\dot{S}_{\rm vN} \geq 0$ and tends to *erase* fine–tuned negative–energy fluxes on times $\gtrsim \tau_{\rm lock}$.

Conclusion. Traversable, macroscopic wormholes require ANEC violation exceeding QEI bounds over scales $\gg \xi$ (correlation length); this is *not* achievable within the admissible spectral parameter space. Microscopic throats of size $\sim \xi$ may occur as transient, nontraversable defects.

16.5 Chronology Protection and Global Hyperbolicity

Attempted creation of closed timelike curves (CTCs) forces a *chronology horizon*. For Hadamard states on nearly–CTC geometries, the renormalized stress tensor exhibits instabilities (vacuum polarization blow-up) that drive backreaction:

$$\langle T_{\mu\nu}\rangle_{\rm ren} \;\to\; \infty \quad \text{as the chronology horizon is approached.} \tag{16.12}$$

In the spectral open dynamics, CP–divisible Lindblad evolution (15.12) *requires* a globally hyperbolic domain; otherwise, no consistent Markovian generator exists that preserves causality and positivity. Together with the foundational principle "no event before time," this yields a *spectral chronology protection*: CTC assemblies either (i) cannot be reached within Gates (S/G/D), or (ii) trigger runaway backreaction that restores global hyperbolicity.

16.6 Diagnostics and Observational Windows

- **Ringdown and echoes:** constraints on ϵ_{gap} and reflectivity $\mathcal{R}$ via late–time GW tails [Eq. (16.9)].
- **Tidal Love numbers:** $k_2 \neq 0$ for spectral solitons; BH baseline $k_2 = 0$.
- **Shapiro delay and lensing:** precise time–delay tests constrain $\Phi(r)$ and photon–sphere location; consistency with Gate (G) bounds on c_T.
- **Pulsar timing in binaries:** periastron advance and Lense–Thirring from $\Phi(r), m(r)$ map to spectral EoS parameters; stochastic backgrounds bound ultracompact populations.

16.7 Gates Summary for Strong–Field Sector

(S) Microcausality and front velocities within the g_{eff} cone; no superluminal c_T, c_s.

(G) Hyperbolicity and no–ghost: positive kinetic blocks; stability of radial modes (16.8).

(TQ) Hadamard states, finite $\langle T_{\mu\nu}\rangle_{\text{ren}}$, QEIs obeyed; area/entropy balance at horizons.

(D) Nonnegative entropy production; viscous/open corrections damp ANEC–violating pulses and forbid stable CTCs.

Summary. Within the spectral framework, classical black holes are recovered as stationary Killing–horizon solutions satisfying Gates (S/G/TQ/D). Horizonless ultracompact *spectral solitons* may mimic some observables (light rings, echoes) while remaining consistent with causality and stability. Traversable macroscopic wormholes and time machines are excluded by a combination of energy–condition bounds (quantum and open–system) and the necessity of globally hyperbolic, CP–divisible dynamics. The sector yields concrete, falsifiable diagnostics in ringdown, Love numbers, and precision timing.

Chapter 17

Global Parameter Maps: From Spectral Couplings to Observables

This chapter assembles a *single reference* that maps the structural/spectral parameters of the Quantum–Aether framework to laboratory, astrophysical, and cosmological observables. It complements the likelihood pipelines of Vol. II, Chap. **??** and the prior construction of App. **??** by providing explicit leading–order formulas, gate–aware validity domains, and Jacobians for error propagation.

17.1 Scope and Canonical Parameter Vector

We collect the minimal *spectral* parameter set into

$$\boldsymbol{\theta}_{\rm spec} = \Big(M_{\rm eff},\ c_T,\ c_{s,\parallel}, c_{s,\perp},\ Z_\perp, \Delta Z,\ \eta_0,\ \Gamma_\eta,\ \hat{\chi}^{(0)},\ \{v_\eta\},\ g_{\eta\Phi}, h_{\eta S},\ \xi,\ \dots \Big), \tag{17.1}$$

where $M_{\rm eff}$ is the state–dependent Planck scale, c_T the tensor speed, $c_{s,\parallel/\perp}$ directional scalar speeds, $Z_\perp, \Delta Z$ gradient moduli, η_0 background viscosity, Γ_η dissipative rate, $\hat{\chi}^{(0)}$ internal director, $\{v_\eta\}$ potential parameters of V_η, $(g_{\eta\Phi}, h_{\eta S})$ couplings to matter/structural sectors, and ξ a freeze–out (correlation) length. The observable vector is organized by domain,

$$\boldsymbol{\mathcal{O}} = \Big(\boldsymbol{\mathcal{O}}_{\rm PN},\ \boldsymbol{\mathcal{O}}_{\rm EM/QED},\ \boldsymbol{\mathcal{O}}_{\rm scat},\ \boldsymbol{\mathcal{O}}_{\rm CMB/LSS/GW},\ \boldsymbol{\mathcal{O}}_{\rm lens} \Big). \tag{17.2}$$

The map is a (generally scale–dependent) function $\boldsymbol{\mathcal{O}} = \mathbf{F}[\boldsymbol{\theta}_{\rm spec}; k, z]$ with domain restricted by the Gates (S/G/TQ/D).

17.2 Gate–Aware Priors and Validity Domains

We impose hard priors (see Vol. II, Sec. **??** and App. **??**):

$$\textbf{(S)}\ \ v_{\rm front}\Big(c_T, c_{s,\parallel/\perp}\Big) \le 1, \qquad \textbf{(G)}\ \ \mathsf{K} \succ 0,\ \ c_T^2 > 0,\ c_{s,\parallel/\perp}^2 > 0,$$

$$\textbf{(TQ)}\ \text{Hadamard initial data, forward–limit positivity, IR/UV control,} \qquad \textbf{(D)}\ \ \sigma_{\rm diss}(\eta_0, \Gamma_\eta) \ge 0. \tag{17.3}$$

Maps quoted below hold to leading order in small parameters $\{\delta c \equiv c_T - 1,\ \Delta c_s^2 \equiv c_{s,\parallel}^2 - c_{s,\perp}^2,\ \eta_0,\ \xi k\}$.

17.3 Relativistic/PN Tests (Solar–System & Binary Pulsars)

PPN sector. To first order in δc and for weak fields,

$$\gamma_{\rm PPN} - 1 \simeq \alpha_\gamma \, \Delta Z \; + \; \beta_\gamma \, \Delta c_s^2, \qquad \beta_{\rm PPN} - 1 \simeq \alpha_\beta \, \frac{\partial \ln M_{\rm eff}}{\partial \ln \rho}, \tag{17.4}$$

with model–dependent coefficients $\alpha_\gamma, \beta_\gamma, \alpha_\beta$ fixed by the curvature sector (Vol. I, Ch. 3). Shapiro delay parameter is tied to $\gamma_{\rm PPN}$.

GW speed.

$$\alpha_T \equiv c_T^2 - 1 \; \simeq \; \delta c_T(\boldsymbol{\theta}_{\rm spec}), \qquad |\alpha_T| \lesssim 10^{-15} \;\; \text{(multi–messenger bound)}. \tag{17.5}$$

17.4 Electromagnetism/QED Corrections

Effective $U(1)$ and fine structure. Isometries of the internal fiber induce an emergent $U(1)$ with $\alpha_{\rm eff} = \alpha\,(1 + \delta_\alpha)$, where, to leading order,

$$\delta_\alpha \simeq \varsigma_1 \, \Delta Z + \varsigma_2 \, \frac{\eta_0}{\Lambda_{\rm sc}} \; + \; \cdots . \tag{17.6}$$

Lamb shift and $g-2$. Shifts are parameterized as SM prediction plus spectral correction:

$$\Delta E_{\rm Lamb} = \Big[\Delta E_{\rm Lamb}\Big]_{\rm SM}\Big(1 + \lambda_{\rm L}\, \delta_\alpha + \lambda'_{\rm L}\, \Delta c_s^2\Big), \tag{17.7}$$

$$a_\ell = \Big[a_\ell\Big]_{\rm SM}\Big(1 + \lambda_a\, \delta_\alpha + \cdots\Big), \qquad \ell = e, \mu. \tag{17.8}$$

17.5 Scattering & Forward Positivity

LSZ map. Residues of two–point functions fix wavefunction renormalizations $\mathcal{Z}_i(\boldsymbol{\theta}_{\rm spec})$. The forward amplitude $A(s, t\to 0)$ obeys

$$\frac{\partial^2}{\partial s^2} A(s,0)\Big|_{s=s_0} \geq 0 \quad \Rightarrow \quad \sum_n \alpha_n(\text{Wilson})\, \theta_n(\boldsymbol{\theta}_{\rm spec}) \geq 0, \tag{17.9}$$

enforcing inequalities among $(Z_\perp, \Delta Z, c_T, c_s)$.

17.6 Cosmology: $(n_s, r, f_{\rm NL}, g_*, \Delta N_{\rm eff}, w_{\rm eff})$

During η–wave inflation (Vol. II, Chap. **??**),

$$n_s - 1 \simeq -2\epsilon_\eta - \eta_\eta - s_c(\Delta c_s), \qquad r \simeq 16\, \epsilon_\eta \, \frac{c_s}{c_T}, \tag{17.10}$$

$$f_{\rm NL}^{(2)} \sim \mathcal{O}\!\left(\frac{1 - c_s^2}{c_s^2}\right) + \mathcal{O}(\text{anisotropic mixing}), \qquad g_* \propto \Delta c_s^2 \, (\hat{k}\cdot\hat{\chi}^{(0)})^2. \tag{17.11}$$

Reheating map (Vol. II, Sec. **??**):

$$T_{\rm RH} = T_{\rm RH}(\Gamma_\eta, g_{\eta\Phi}, h_{\eta S}), \qquad \Delta N_{\rm eff} = \Delta N_{\rm eff}\Big(\text{branchings}, T_{\rm RH}\Big). \tag{17.12}$$

Late–time acceleration (Vol. II, Chap. **??**): $w_{\rm eff}(z) = -1 + \delta(z; \eta_0, \{v_\eta\})$.

17.7 Lensing, Time Delays, and EHT

Deflection and delay are set by the post–Newtonian potential Φ and c_T:

$$\hat{\alpha}(b) \simeq \frac{4GM(b)}{b}\Big(1+\delta_\gamma\Big), \qquad \delta_\gamma \approx \gamma_{\rm PPN}-1, \tag{17.13}$$

$$\Delta t_{\rm Shapiro} \simeq (1+\gamma_{\rm PPN})\int \frac{2\Phi}{c_T^3}\, dl. \tag{17.14}$$

Shadow diameter $d_{\rm sh} \approx 6\sqrt{3}\,M\,[1+\mathcal{O}(\delta_\gamma,\delta c_T)]$ with subleading dependence on anisotropy through ΔZ.

17.8 Gravitational Waves: Propagation and Ringdown

Propagation on large scales (App. **??**):

$$\omega^2 = c_T^2 k^2 - i\,\nu_T k^2 + \cdots, \qquad \nu_T = \nu_T(\eta_0,\Gamma_\eta), \tag{17.15}$$

with $|\delta c_T| \ll 1$, $\nu_T \geq 0$ (Gate D). Ringdown spectra depend on compact–object map (Vol. III, Chap. 16); echo delay $\Delta t_{\rm echo}$ ties to $\epsilon_{\rm gap}$ via Eq. (16.9).

17.9 Defects, LSS Morphology, and Maps to Statistics

Kibble–Zurek density (App. **??**):

$$n_{\rm def} \sim \xi^{-d} \;\propto\; \Big|\dot{\lambda}_0\Big|^{d\nu/(1+\nu z)} \Big(\det \mathbf{D}_{\parallel,\perp}\Big)^{+1/2}, \tag{17.16}$$

with anisotropic diffusivities $\mathbf{D}$ from $\{Z_\perp, \Delta Z\}$. Minkowski functionals/skeleton coefficients inherit the mode–interference kernel $\mathcal{I}(x)$ (Vol. II, Chap. **??**).

17.10 Jacobians, Fisher Information, and Error Propagation

Define the Jacobian $\mathbf{J} \equiv \partial\boldsymbol{\mathcal{O}}/\partial\boldsymbol{\theta}_{\rm spec}$. For a Gaussian posterior in the observables,

$$\mathbf{F}_{\rm spec} \;=\; \mathbf{J}^{\top}\,\mathbf{C}_{\rm obs}^{-1}\,\mathbf{J} \quad\Rightarrow\quad \mathbf{C}_{\rm spec} \approx \mathbf{F}_{\rm spec}^{-1}, \tag{17.17}$$

with gate–aware reweighting (hard priors as constraints, soft priors from App. **??**). For non–linear maps, use Gauss–Newton or adjoint differentials from the numerical pipelines (Vol. II, Chap. **??**).

17.11 Canonical Tables (Quick Reference)

Table 17.1: Spectral → Observable map (leading order) with gates and current bounds.

Spectral parameter	**Observable(s)**	**Map (LO)**	**Gate / Bound**
$M_{\rm eff}$	$G_{\rm eff}=1/M_{\rm eff}^2$, PN β	$\beta_{\rm PPN}-1\simeq\alpha_\beta\,\partial_{\ln\rho}\ln M_{\rm eff}$	(G), Cassini/LLR
c_T	GW speed α_T	$\alpha_T=c_T^2-1$	(S), $\lvert\alpha_T\rvert\lesssim10^{-15}$
$c_{s,\parallel/\perp}$	g_*, bispectrum anisotropy	$g_*\propto\Delta c_s^2(\hat{k}\cdot\hat{\chi})^2$	(G), CMB $\lvert g_*\rvert\ll1$
$Z_\perp,\Delta Z$	$\gamma_{\rm PPN}$, lensing	$\gamma-1\simeq\alpha_\gamma\Delta Z+\beta_\gamma\Delta c_s^2$	(S/G), Cassini
η_0,Γ_η	damping ν_T, $T_{\rm RH}$	$\nu_T=\nu_T(\eta_0,\Gamma_\eta)$; $T_{\rm RH}=T_{\rm RH}(\Gamma_\eta,\cdots)$	(D), BBN/Planck
$\{v_\eta\}$	$(n_s,r,f_{\rm NL})$	Eqs. (17.10)–(17.11)	(TQ), CMB
$g_{\eta\Phi},h_{\eta S}$	$\Delta N_{\rm eff}$	Eq. (17.12)	BBN/CMB
$\hat{\chi}^{(0)}$	directional stats	$g_*\propto(\hat{k}\cdot\hat{\chi}^{(0)})^2$	(S/G)
ξ	$n_{\rm def}$	$n_{\rm def}\sim\xi^{-d}$ (App. **??**)	LSS/strings
δ_α	Lamb, $g-2$	Eqs. (17.7)–(17.8)	QED precision
$\epsilon_{\rm gap}$	ringdown echoes	$\Delta t_{\rm echo}\sim2M\ln(1/\epsilon_{\rm gap})$	GW catalogs

17.12 Analyst's Recipe

1. **Select regime** (PN, CMB, LSS, GW, QED) and enforce the gates in Eq. (17.3).
2. **Evaluate the map $\mathcal{O}=\mathbf{F}[\boldsymbol{\theta}_{\rm spec}]$** using the domain–specific formulas above (and the numerical kernels of Vol. II, Chap. **??**).
3. **Propagate errors** with the Jacobian (Eq. (17.17)); overlay hard/soft priors from App. **??**.
4. **Report** $(\boldsymbol{\theta}_{\rm spec},\mathbf{C}_{\rm spec})$ with gate compliance flags and dominant-observable sensitivities (columns of $\mathbf{J}$).

Summary. The tables and formulas in this chapter turn the spectral model into a *working interface* with data: each coupling or structural descriptor has a clear observational footprint, bracketed by the consistency gates and present bounds, and ready for forecasting with next–generation surveys.

Part IV

Synthesis, Limits, and Final Maps

Chapter 18

What We Recovered and Where It Breaks

abstract

We summarize how the spectral medium reproduces classical and quantum physics, where small, testable deviations are expected, and which domains are provably beyond the effective description. The chapter closes with *kill criteria*: operational rules under which the model—given the consistency gates S/G/TQ/D— is ruled out by data.

18.1 Classical Physics: Recovery Benchmarks

- **Mechanics & inertia.** Rest mass from spectral energy: $mc^2 = \int d^3x\, \varepsilon_{\text{spec}}, \quad E^2 = (pc)^2 + (mc^2)^2$ (Vol. III, Chap. **??**).
- **Conservation laws.** Noether charges for translations/phase $\Rightarrow$ momentum/charge conservation; continuity equations close once D-gate (dissipation) is enforced (Vol. III, Sec. **??**).
- **Relativistic limit.** Emergent Lorentz sector with universal c from RG flow under small η: $c_T \to c,\ c_{s,\parallel/\perp} \to c$ (Vol. III, Sec. 3.4). PPN parameters reduce to GR in the isotropic, weak-field limit.
- **Geometric/optical laws.** Fermat's principle on g_{eff}, lensing, Shapiro delay; controlled birefringence when $\Delta Z \neq 0$.

18.2 Quantum Physics: Recovery Benchmarks

- **Hilbert space from $\mathcal{L}^{(2)}$**: symplectic form Ω, complex structure J, inner product $\langle \cdot, \cdot \rangle$ (Vol. III, Sec. 1.3).
- **Damascus interpretation.** Measurements are *spectral flooding* into stable pointer modes; Born's rule is a stationary energy-share limit with controlled corrections (Sec. 1.5).
- **Entanglement as structure.** Nonseparability along χ^μ-threads, Tsirelson saturation, and no-signalling via causal, CP Lindblad generators (Sec. 1.6).

18.3 Predictive Deviations & Stress Points

1. **Mild anisotropy** in scalar/tensor propagation: $c_{s,\parallel} - c_{s,\perp},\ \delta c_T \Rightarrow g_*$ and angle-weighted bispectra (links to Vol. II).
2. **Colored Zeno** and phase-only double-slit shifts: decoherence spectra imprint beyond textbook collapse (Sec. 1.7).
3. **GKSL-consistent radiation reaction:** no runaways; damping rates tied to η, Γ (Sec. **??**).

18.4 Controlled Domains and Where It Breaks

Validity (IR/EFT). $k\xi \ll 1$, weak anisotropy $|\Delta Z| \ll Z_\perp$, subluminality, $\mathsf{K} \succ 0$, and forward-positivity (TQ).

Breakdown modes. (i) Near-core, high-k solitonic structure; (ii) strong anisotropy with dispersion-cone splitting beyond observational bounds; (iii) regimes where fitting data would require $c_T > 1$ or negative kinetic blocks— both violate S/G gates.

18.5 Kill Criteria (Operational)

We define *tiered* kill criteria. If any tier is met by robust data, the model is considered falsified.

Tier A (Gate violation). Best-fit to any dataset combination requires violating at least one gate:

(S) $v_{\text{front}} > 1$; (G) K non-positive or $c_{s,T}^2 < 0$; (TQ) forward positivity fails; (D) $\sigma_{\text{diss}} < 0$.

This tier is *absolute*.

Tier B (Incompatible cross-relations). Measured observables fall outside all parameter images $\mathbf{F}[\boldsymbol{\theta}_{\text{spec}}]$ under the gates, e.g.

(B1) (r, n_s, g_*) violating the single-clock η-wave inequalities derived in Vol. II, Chap. **??**; (18.1)

(B2) Joint QED shifts (Lamb, $g-2$) requiring δ_α incompatible with $(Z_\perp, \Delta Z, \eta_0)$ under positivity. (18.2)

(B3) Multi-messenger GW constraints demanding $|c_T - 1|$ beyond the gated envelope. (18.3)

Tier C (Practical null). Global, gate-aware inference (Vol. II, Chap. **??**) yields a posterior confined to a trivial corner that is observationally indistinguishable from GR+SM *and* leaves no viable parameter volume for the distinct spectral signatures (e.g., g_*, colored-Zeno windows, GKSL damping). Formally:

$$\text{Vol}\left(\{\boldsymbol{\theta}_{\text{spec}} :\ \mathbf{F} \text{ yields a detectable spectral signature in any planned survey}\}\right) \approx 0.$$

When Tier C holds across the next-generation program, we treat the framework as *empirically exhausted*.

□

Appendix I: Final Spectral-to-Observable Tables

Post-Newtonian (PN) & Relativistic Tests

Observable	Spectral map (LO)	Gate / Bound
$\gamma_{\rm PPN} - 1$	$\alpha_\gamma\,\Delta Z + \beta_\gamma\,\Delta c_s^2$	(S,G) Cassini
$\beta_{\rm PPN} - 1$	$\alpha_\beta\,\partial_{\ln\rho}\ln M_{\rm eff}$	(G) LLR
$\lvert c_T^2 - 1\rvert$	$\delta c_T(\boldsymbol{\theta}_{\rm spec})$	(S) $\lesssim 10^{-15}$
Shapiro delay	$(1+\gamma_{\rm PPN})\int 2\Phi/c_T^3\,dl$	(S) Solar System
Perihelion precession	PN series in $(M_{\rm eff}, \Delta Z)$	(S,G) Mercury

Wave Speeds: c_T and $c_{s,\parallel/\perp}$

Sector	Speed (LO)	Gate / Bound
Tensor	$c_T^2 = 1 + \delta c_T(\eta_0, \Delta Z, \ldots)$	(S) $\ll 1$
Scalar (avg.)	$c_s^2 \simeq c^2\,[1 + \mathcal{O}(\Delta Z)]$	(G) > 0
Anisotropy	$c_{s,\parallel}^2 - c_{s,\perp}^2 = \Delta c_s^2(\hat{\chi}^{(0)})$	CMB g_*

QED Precision: Lamb, $g-2$, Fine Structure

Quantity	Spectral correction	Gate / Bound
$\alpha_{\rm eff}$	$\alpha(1+\delta_\alpha),\ \delta_\alpha = \varsigma_1\Delta Z + \varsigma_2\eta_0/\Lambda_{\rm sc}$	(TQ)
Lamb shift	$\Delta E_{\rm Lamb} = \left[\Delta E\right]_{\rm SM}(1 + \lambda_{\rm L}\delta_\alpha + \lambda'_{\rm L}\Delta c_s^2)$	Spectroscopy
$g-2$	$a_\ell = \left[a_\ell\right]_{\rm SM}(1 + \lambda_a\delta_\alpha)$	(e,μ) data

Scattering Templates & Positivity

Handle	Map/Constraint	Gate / Bound
Field residues	$\mathcal{Z}_i(\boldsymbol{\theta}_{\rm spec})$	LSZ
Forward limit	$\partial_s^2 A(s,0) \geq 0$	(TQ) positivity
Total rates	Optical theorem $\Leftrightarrow$ GKSL damping	(D)

CMB/LSS: $(n_s, r, f_{\rm NL}, g_*, \Delta N_{\rm eff})$

Observable	Spectral map (LO)	Gate / Bound
$n_s - 1$	$-2\epsilon_\eta - \eta_\eta - s_c(\Delta c_s)$	(TQ) CMB
r	$16\,\epsilon_\eta\, c_s/c_T$	B-mode
$f_{\rm NL}^{(2)}$	$\sim (1-c_s^2)/c_s^2 +$ anisotropic mixing	Bispectrum
g_*	$\propto \Delta c_s^2(\hat{k}\cdot\hat{\chi}^{(0)})^2$	$\lvert g_*\rvert \ll 1$
$\Delta N_{\rm eff}$	From $(\Gamma_\eta, \text{branchings}, T_{\rm RH})$	BBN/Planck

Lensing & GW (Propagation/Ringdown)

Quantity	Map (LO)	Gate / Bound
Deflection $\hat{\alpha}$	$4GM/b\,(1+\delta_\gamma)$, $\delta_\gamma \simeq \gamma_{\rm PPN} - 1$	(S,G)
Shapiro delay	$(1+\gamma_{\rm PPN})\int 2\Phi/c_T^3\, dl$	(S)
GW propagation	$\omega^2 = c_T^2 k^2 - i\nu_T k^2 + \cdots$	(D) $\nu_T \geq 0$
Ringdown/echo	$\Delta t_{\rm echo} \sim 2M\ln(1/\epsilon_{\rm gap})$	Catalog fits

Part V

Roadmap Beyond Volume III

Chapter 19

Volume IV — Spectral Particle Physics

abstract

We outline a program to reconstruct particle physics from internal-fiber isometries of the spectral medium. The goals are: (i) derive emergent gauge symmetries and Ward identities; (ii) implement anomaly matching à la 't Hooft; (iii) generate masses and mixings (including a spectral see–saw for neutrinos); (iv) realize effective strong dynamics (confinement, chiral symmetry breaking) with quantitative hadronic matching.

19.1 Emergent Gauge Symmetries from Fiber Isometries

- **Isometries** of the internal spectral fiber $\mathcal{F} \Rightarrow$ a compact Lie group G_{spec} with connection $\mathcal{A}_\mu$ pulled back by D_μ; identify the $U(1) \times SU(2) \times SU(3)$ sector as a maximal subgroup in a symmetry-broken phase of $\mathcal{F}$.
- **Emergent Ward identities:** Noether currents from fiber isometries yield Ward–Takahashi identities for correlators built on g_{eff}.
- **Effective BRST/FP:** construct a gauge-fixing functional on $\mathcal{F}$ and derive a *spectral* BRST charge Q_{BRST} ensuring gauge-parameter independence of S–matrix elements.

19.2 't Hooft Matching and Anomaly Accounting

- **Catalogue:** ABJ, global and mixed gauge–gravitational anomalies from spectral fermion content; Chern–Simons/WZW terms as emergent functionals on $\mathcal{F}$.
- **Matching:** show that IR composites reproduce UV anomaly polynomials; derive cancellation conditions mapping to spectral charges and representations.

19.3 Masses and Mixing

- **Effective Higgs:** a structural resonance in Φ (and density ρ) acts as an order parameter; compute fermion masses via overlap integrals on $\mathcal{F}$.
- **Neutrino masses (spectral see–saw):** heavy modes in the internal fiber generate $m_\nu \simeq -m_D^\top M_R^{-1} m_D$ with M_R set by topological gaps.
- **Flavor matrices:** CKM/PMNS from winding/intersection numbers of internal cycles; predict textures and CP phases with uncertainty bands.

19.4 Effective Strong Dynamics

- **Confinement & χSB:** spectral Skyrme/Faddeev terms realize baryons as solitons; derive GOR relation and leading hadron spectrum.
- **Matching:** map low-energy constants to spectral couplings $(Z_\perp, \Delta Z, M_{\text{eff}})$ under TQ (positivity) gates.

Chapter 20

Volume V — Spectral Quantum Foundations

abstract

We complete the Damascus interpretation: Born's rule as a stationary energy-share limit under spectral flooding; measurement as GKSL-consistent open dynamics; entanglement as structural nonseparability; and spectral quantum control with colored noise and Zeno engineering.

20.1 Damascus Interpretation: Compact Proof

- **Hilbert space** from $\mathcal{L}^{(2)}$; pointer sectors stabilized by large (ρ, η); **Born** weights from long-time energy partition.
- **Bounds:** quantify controlled deviations for finite locking rates.

20.2 Entanglement, No-Signalling, and Threads

- **χ^{μ}-threads:** graph-based bipartitions; Tsirelson saturation; GKSL locality $\Rightarrow$ no superluminal signalling.

20.3 Spectral Quantum Control

- Colored Zeno; noise spectroscopy; protocols for cold atoms and waveguides.

Chapter 21

Volume VI — Spectral Matter Sciences

abstract

We extend the framework to chemistry and condensed matter: atoms as quantized spectral resonators; bonds as synchronization; topological phases and constrained dynamics; and a prelude to spectral biology as large-scale coherent resonance.

21.1 Spectral Chemistry

- Periodic table as a topological map of stable resonance patterns on $\mathcal{F}$; selection rules from fiber symmetries.

21.2 Condensed-Matter Phenomena

- Topological phases; fracton-like constrained mobility from internal anisotropy; viscous–elastic (viscoelastic) spectral phase transitions.

21.3 Prelude to Spectral Biology

- Global coherent resonance as a working definition of the living state.

Chapter 22

Post-V3 Experimental Program

22.1 CMB/LSS

- Directional bispectrum template (linked to $c_s(\theta)$ anisotropy); constraints on g_*; defect/resonance imprints in Minkowski functionals and skeleton statistics.

22.2 Gravitational Waves

- Bounds on c_T and structural damping/viscosity via LIGO/Virgo/KAGRA/PTA; ringdown/echo phenomenology for horizonless solitonic alternatives.

22.3 Precision Labs

- Fine-structure and Lamb shifts; $(g-2)$; map $\delta\alpha$, Δc to spectral parameters under TQ gates.

22.4 Table-top/Guided Experiments

- Phase-only tagging in double-slit; colored Zeno in guided optics or cold atoms.

Chapter 23

Decisive Discriminants vs. ΛCDM/QFT

- **Directional bispectrum template** not reproducible by single-field, isotropic EFTs.
- **Environment correlations** between late-time $w_{\text{eff}}(t)$ and lensing/LSS.
- **GW damping** with mild, structured c_T departures tied to background fields.

Chapter 24

Global Kill Criteria (Post-V3)

1. **Causality/Speeds:** persistent constraints forcing $c_T = 1$ and $\Delta c_s^2 \to 0$ across redshift/environments within errors that exclude the entire gated parameter volume.
2. **Non-Gaussianity:** null detection of the directional bispectrum at next-generation sensitivity excluding the anisotropic window under S/G/TQ/D gates.
3. **No spectral footprint** in scattering/QED precision that contradicts the spectral maps *while* fitting cosmology, closing the parameter space.

Chapter 25

Near-Term Deliverables (During V3⇒V4)

- Full $f_{\rm NL}$ file + calibrated directional template.
- $(\Gamma_\eta, \kappa_\eta) \mapsto \{T_{\rm RH}, \Delta N_{\rm eff}\}$ maps with BBN/CMB shading.
- RG phase/flow diagram with S/G/TQ/D gates overlaid.
- Compact "Emergence of Laws" appendix to close the conceptual loop.